

# Asset Encumbrance, Bank Funding and Fragility\*

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## Abstract

We propose a model of asset encumbrance by banks subject to rollover risk and study the consequences for fragility, funding costs, and prudential regulation. A bank's choice of encumbrance trades off the benefit of expanding profitable investment funded by cheap long-term secured debt against the cost of greater fragility due to unsecured debt runs. We derive several testable implications about privately optimal encumbrance ratios. Deposit insurance or wholesale funding guarantees induce excessive encumbrance and exacerbate fragility. We show how regulations such as explicit limits on encumbrance ratios and revenue-neutral Pigouvian taxes can mitigate the risk-shifting incentives of banks.

**Keywords:** asset encumbrance, rollover risk, wholesale funding, fragility, runs, secured debt, unsecured debt, encumbrance limits, encumbrance surcharges.

**JEL classifications:** G01, G21, G28.

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# 1 Introduction

A bank attracts long-term secured funding by encumbering assets to creditors who retain an exclusive claim to these pledged assets upon bank default. Examples of such secured debt instruments include covered bonds, term repurchase agreements, mortgage-backed securities, and collateralised debt securities. In section 1.1, we document three facts about asset encumbrance: the ratios of encumbered-to-total assets are large, have grown over time, and are heterogeneous across banks and countries.

The growing importance of long-term secured funding raises questions about the relationship between asset encumbrance, bank fragility, and the role of public policy. What are the benefits and costs of asset encumbrance in relation to bank fragility? How do economic and financial factors influence a bank's choice of asset encumbrance? How is this encumbrance choice affected by policies aimed at promoting financial stability, such as deposit insurance or wholesale funding guarantees? If these policies have unintended consequences, how should corrective measures be designed?

In section 2, we propose a model of asset encumbrance that focuses on the interaction between long-term secured debt and demandable unsecured debt. Building on [Rochet and Vives \(2004\)](#) and [Vives \(2014\)](#), banks are subject to rollover risk in unsecured debt markets. Banks with initial equity seek funding for profitable but illiquid long-term investment. Debt is issued in two segmented markets. Banks attract unsecured funding from risk-neutral investors and secured funding from risk-averse investors by ring-fencing assets into a bankruptcy-remote entity. Secured debt is insulated from shocks to a bank's balance sheet that are borne by unsecured debtholders.<sup>1</sup>

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<sup>1</sup>The Modigliani-Miller theorem fails in our model. The presumed segmentation of funding markets according to risk preferences of investors breaks the irrelevance between secured and unsecured debt. Moreover, costly liquidation of investment breaks the irrelevance between debt and equity.

In section 3, we use global games techniques (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005) to uniquely pin down the equilibrium when precise private information arrives before the rollover decision. As a result, a run on unsecured debt occurs if, and only if, the shock exceeds a threshold given by the value of unencumbered assets. We link the incidence of runs to the bank's choice of asset encumbrance and solve for the face values of secured and unsecured debt.

Asset encumbrance alters run dynamics by driving a wedge between the conditions for illiquidity and insolvency. If a bank must prematurely liquidate assets to satisfy unsecured debt withdrawals, it can only use unencumbered assets since encumbered assets are pledged to secured debtholders. But if unsecured debt is rolled over, the bank can pay unsecured debtholders using residual encumbered assets once secured debtholders have been paid. This is possible because of 'over-collateralization', whereby the value of encumbered assets is greater than the face value of secured debt. While illiquidity of the bank depends only on unencumbered assets, insolvency depends on all assets. We show that the illiquidity condition is more binding than the insolvency condition if unsecured debt is cheap. Asset encumbrance can, therefore, make solvent banks illiquid and prone to unsecured debt runs. This result contrasts with Rochet and Vives (2004), where an illiquid bank is always insolvent.

Greater asset encumbrance ratios induce two opposing effects on bank fragility. The benefit of greater encumbrance is to raise a greater amount of cheap secured debt to finance profitable investment. The potential cost is that too few unencumbered assets are available to meet unsecured debt withdrawals, thereby exacerbating rollover risk. This cost becomes material once the bank encumbers more than one unit of assets to raise one unit of long-term secured funding and depends on the cost of recovering encumbered assets after bank failure. These costs are likely to be large. Secured debtholders may have to assert their claim against unsecured debtholders in

costly and protracted legal proceedings (Duffie and Skeel, 2012; Fleming and Sarkar, 2014; Ayotte and Gaon, 2011). Access to critical infrastructure, such as risk management systems, may also be disrupted after bank failure, reducing the realized value of encumbered assets (Bolton and Oehmke, 2016). As a result, the net effect of greater asset encumbrance is to exacerbate run risk.

The model yields several testable implications reviewed in section 4. A bank's privately optimal encumbrance ratio balances marginal cost (greater fragility and a lower probability of surviving a run) against marginal benefit (greater profitability conditional on survival). Accordingly, greater encumbrance ratios or, equivalently, higher secured funding shares arise when (a) investment is more profitable; (b) funding costs are lower; (c) the distribution of shocks to the balance sheet is more favorable; (d) conditions in unsecured funding markets are benign; (e) the costs of recovering encumbered assets are low; and (f) liquidation values of investment are high. Although the relationship between encumbrance and bank capital is non-monotonic in general, tighter predictions arise for specific distributional assumptions. These implications are consistent with existing evidence and can inform future empirical work.

In section 5, we study normative implications of asset encumbrance. Our notion of welfare is constrained efficiency, whereby a planner takes the rollover risk and incomplete information in unsecured debt markets as given when choosing the socially optimal encumbrance ratio. In many countries, unsecured debtholders enjoy the benefits of public guarantee schemes. While such privileges usually apply to retail depositors, they are often extended to unsecured wholesale depositors during financial crises. Although guarantees mitigate bank fragility ex post, there can be excessive risk-taking ex ante. Since banks fail to internalize the cost of servicing the guarantee, they have incentives to excessively encumber assets, thereby exacerbating fragility.<sup>2</sup>

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<sup>2</sup>While we do not explicitly model deposit insurance premiums, these are typically insensitive

Our model is a natural framework to explore different prudential regulations. If a planner can control asset encumbrance ex ante, limits on encumbrance ratios achieve constrained efficiency. If asset encumbrance ratios can only be observed ex post, revenue-neutral Pigouvian taxes on asset encumbrance ratios achieve the social optimum. A linear tax on encumbrance corrects the incentives of a bank not to shift risk to the guarantor, while a lump-sum rebate of the revenue generated ensures that the bank has enough resources to avoid excessive fragility.

In light of these normative implications, we review and evaluate regulatory measures across several jurisdictions aimed at curbing asset encumbrance. While many countries have adopted a cap on encumbered assets, the United States has implemented a policy to cap the share of secured debt to total liabilities. In the context of our model, both measures are equivalent. Interestingly, the asset encumbrance caps for Italian banks are contingent on their capital ratios. Because of the non-linear relationship between encumbrance and bank capital, this policy may be ineffective. In the Netherlands, the encumbrance limit is regularly evaluated over the cycle.

In section 6, we relax some assumptions to derive an additional testable implication and to explore the limits of our model. First, we consider a risk-premium earned by risk-neutral investors. We show that higher risk premiums decrease asset encumbrance ratios. Second, we allow risk-averse investors to have limited wealth. Consequently, a bank's choice of asset encumbrance may not fully trade-off the benefit of cheap secured funding versus the cost of higher fragility. Third, we consider noisy private information about the balance sheet shock. As a result, there is a range of shocks for which the bank is illiquid but solvent and would benefit from liquidity support. Section 7 concludes. All proofs are in the Appendix.

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to encumbrance ratios. As a result, even if premiums are fairly priced, a bank has incentives to encumber assets excessively.

**Literature.** Our paper contributes to a literature on bank runs where the unique equilibrium is pinned down using global games (Morris and Shin, 2001; Goldstein and Pauzner, 2005; Eisenbach, 2017). In particular, we build on Rochet and Vives (2004) where unsecured debtholders delegate their rollover decisions to professional fund managers, so the decisions to roll over debt are global strategic complements and a run is the consequence of a coordination failure as a bank’s fundamentals deteriorate.<sup>3</sup> Our contribution is to introduce secured funding and to identify how asset encumbrance affects both the run risk and the pricing of unsecured debt.

Our paper adds to the nascent literature on the interaction between secured and unsecured debt. In a corporate finance setting, Auh and Sundaresan (2015) examine how short-term secured debt interacts with long-term unsecured debt. Ranaldo et al. (2017) study short-term secured and unsecured debt in money markets, where shocks to asset values lead to mutually reinforcing liquidity spirals. Our focus, by contrast, is on the interaction between long-term secured and demandable unsecured debt.

Finally, our paper contributes to the debate on the financial stability implications of asset encumbrance. Gai et al. (2013) and Eisenbach et al. (2014) develop partial equilibrium models of the interplay between secured and unsecured funding. Gai et al. (2013) show that interim liquidity risk and asset encumbrance intertwine and can generate a ‘scramble for collateral’ by short-term secured creditors. Eisenbach et al. (2014) examine a range of wholesale funding arrangements using a balance sheet approach with exogenous creditor decisions. Their model suggests that asset encumbrance increases insolvency risk when the encumbrance ratio is sufficiently high.<sup>4</sup>

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<sup>3</sup>Goldstein and Pauzner (2005) study one-sided strategic complementarity due to the sequential service constraint of banks (Diamond and Dybvig, 1983). Matta and Perotti (2017) contrast the sequential service constraint with mandatory stay of illiquid assets and study its impact on run risk. Eisenbach (2017) shows that rollover risk due to demandable debt effectively disciplines banks when they are subject to idiosyncratic shocks but a two-sided inefficiency arises for aggregate shocks.

<sup>4</sup>Hardy (2014) studies the effect of asset encumbrance on bank resolution policy. Helberg and Lindset (2014) studies the link between encumbrance and bank capital in a model of default risk.

## 1.1 Stylized facts

To set the stage for our theoretical analysis, we document three facts about the shares of secured funding and the associated ratios of asset encumbrance of banks.

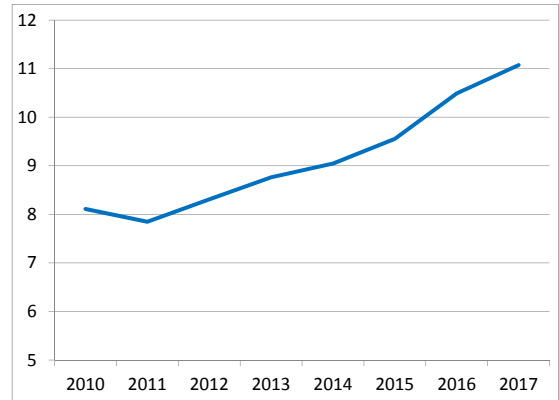
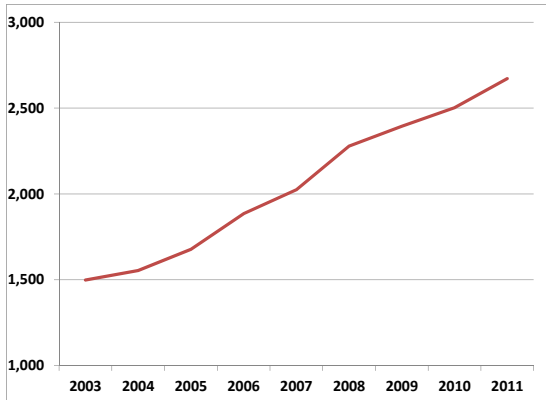
**Fact 1: Secured debt is a sizable portion of bank funding.** The share of outstanding secured debt to total liabilities for Euro Area banks was 33%, or roughly 1.3 trillion US dollars, as of July 2013. For US banks, the corresponding share was 10%, equivalent to 200 billion US dollars (IMF, 2013).<sup>5</sup>

**Fact 2: Secured funding shares of banks have grown.** Figure 1a shows the total global issuance of covered bonds. Its stock has almost doubled during 2003–11, a period for which comparable data is available. ECB (2016) documents a similar increase in the secured funding share for euro area banks for the period 2005-15. Similarly, secured funding (comprising repos and covered bonds) for Canadian banks rose from 8% to 11% of total bank debt funding between 2010-17 (Figure 1b).

**Fact 3: Encumbrance ratios are heterogeneous across banks and countries.** Asset encumbrance ratios vary widely across countries (Figure 1c) and across banks. In a recent survey, EBA (2016) documents that asset encumbrance ratios for a sample of about 200 European banks in 2015 varied between zero to over 65%, with an average of 26% and interquartile range of 20%. The secured funding shares of Canadian banks in 2016 exhibit a large degree of heterogeneity as well. Similarly, CGFS (2013) and Juks (2012) document a large degree of heterogeneity in encumbrance ratios.

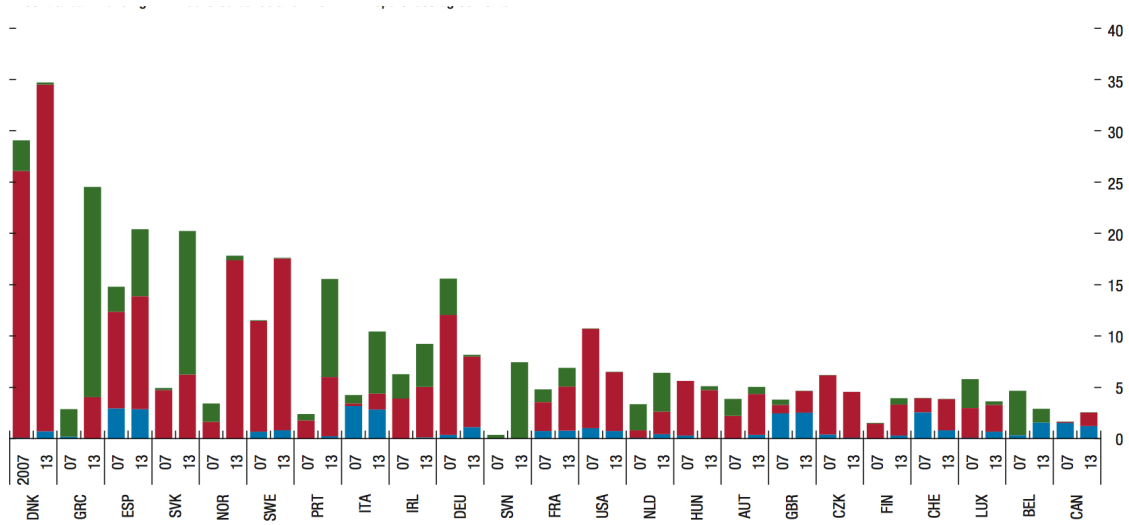
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<sup>5</sup>Across a sample of European and North American banks, the average asset encumbrance ratio, as a percentage of total bank assets, was 15% in 2013. Encumbered assets were primarily being used as collateral for asset-backed securities, covered bonds, and repurchase agreements ('repos').



(a) Total global stock of covered bonds

(b) Secured funding shares in Canada



(c) Asset encumbrance ratios across jurisdictions in 2007 and 2013

Figure 1: Asset encumbrance ratios and secured funding shares. Panel (a) shows the global volume of covered bonds (in billion EUR) for a period in which comparable data is available. Panel (b) plots secured funding (covered bonds and repos) as a share of total debt funding (total assets minus equity) for an average of 81 reporting Canadian banks. Panel (c) plots asset encumbrance ratios across several countries in 2007 and 2013. Sources: (a) European Covered Bond Council (ECBC); (b) Office of the Superintendent of Financial Institutions (OSFI) Consolidated Balance Sheet (M4) return template, <http://www.osfi-bsif.gc.ca/Eng/fi-if/rtn-rlv/fr-rf/dti-id/Pages/M4.aspx>; (c) IMF.



## 2 Model

Our model builds on [Rochet and Vives \(2004\)](#) and [Vives \(2014\)](#). There are three dates  $t = 0, 1, 2$ , a single good for consumption and investment, and a large mass of investors. Investors are indifferent between consuming at  $t = 1$  and  $t = 2$ , but differ in their risk preferences: a first clientele is risk-neutral, while a second is infinitely risk-averse. The latter group can be thought of as pension funds or large institutional investors mandated to hold high-quality and safe assets ([IMF, 2012](#)). Investors receive a unit endowment at  $t = 0$  and may store it until  $t = 2$  at a return  $r > 0$ .

A unit mass of identical risk-neutral bankers has access to profitable investments at  $t = 0$ . These investments mature at  $t = 2$  with return  $R > r$ . Premature liquidation at  $t = 1$  yields a fraction  $\psi \in (0, 1)$  of the return at maturity. Bankers can invest their own funds,  $E \geq 0$ , at  $t = 0$  in order to consume at  $t = 2$ . But bankers can also obtain funding at  $t = 0$  from the segmented investor base by issuing unsecured demandable debt to risk-neutral investors and secured debt to risk-averse investors.<sup>6</sup>

An exogenous amount of unsecured debt,  $U \equiv 1$ , can be withdrawn at  $t = 1$  or rolled over until  $t = 2$ . As in [Rochet and Vives \(2004\)](#), the rollover decision is delegated by the investors to professional fund managers. A fund manager's incentive to rollover is governed by the conservatism ratio,  $0 < \gamma < 1$ .<sup>7</sup> The greater the conservatism ratio, the less likely that a manager rolls over unsecured debt.<sup>8</sup> The face value of unsecured debt,  $D_U$ , is independent of the withdrawal date.

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<sup>6</sup>Consistent with much evidence, unsecured debt issued by banks is assumed to be demandable. While we do not seek to offer a microfoundation, demandability of debt arises endogenously as a commitment device to overcome an agency conflict ([Calomiris and Kahn, 1991](#); [Diamond and Rajan, 2001](#)) or to satisfy liquidity needs ([Diamond and Dybvig, 1983](#)). See also [Rochet and Vives \(2004\)](#).

<sup>7</sup>The conservatism ratio,  $\gamma \equiv \frac{c}{b+c} \in (0, 1)$ , stems from managerial compensation. If the bank fails, a manager's relative compensation from rolling over is negative,  $-c < 0$ . Otherwise, the relative compensation is positive,  $b > 0$ .

<sup>8</sup>Reviewing debt markets during the financial crisis, [Krishnamurthy \(2010\)](#) argues that investor conservatism was an important determinant of short-term lending behavior. See also [Vives \(2014\)](#).

Bankers attract secured funding from risk-averse investors by encumbering a proportion  $\alpha \in [0, 1]$  of assets and placing them into a bankruptcy-remote entity.<sup>9</sup> Denote by  $S \geq 0$  the amount of long-term secured funding, and by  $D_S$  its face value at  $t = 2$ . Table 1 illustrates the balance sheet of a representative bank at  $t = 0$  once funding is raised, investment  $I \equiv E + S + U$  is made, and assets are encumbered.

Assets		Liabilities
(encumbered assets)	$\alpha I$	$S$
(unencumbered assets)	$(1 - \alpha)I$	$U$
		$E$

Table 1: Balance sheet at  $t = 0$  after funding, investment, and asset encumbrance.

We suppose that the balance sheet is subject to a shock,  $A \in \mathbb{R}$ , at  $t = 2$ . The shock may enhance the value of assets,  $A < 0$ . But the crystallization of operational, market, credit or legal risks may require writedowns,  $A > 0$ .<sup>10</sup> The shock has a continuous probability density function  $f(A)$  and cumulative distribution function  $F(A)$ , with decreasing reverse hazard rate,  $\frac{d}{dA} \frac{f(A)}{F(A)} < 0$ , to ensure uniqueness of equilibrium. Bankers and investors are protected by limited liability. Table 2 shows the bank's balance sheet at  $t = 2$  for a small shock and when all unsecured debt is rolled over. Since encumbered assets are ring-fenced, the shock affects only unencumbered assets.

Assets		Liabilities
(encumbered assets)	$R\alpha I$	$SD_S$
(unencumbered assets)	$R(1 - \alpha)I - A$	$UD_U$
		$E_2(A)$

Table 2: Balance sheet at  $t = 2$  after a small shock and unsecured debt is rolled over. The value of bank equity at  $t = 2$  is then  $E_2(A) \equiv RI - A - UD_U - SD_S$ .

<sup>9</sup>Asset encumbrance facilitate two goals. First, secured investors have an exclusive claim to the assets set aside. Second, secured investors can lay claim to the assets in all states of the world. In practice, this is achieved by ring-fencing the assets in a bankruptcy-remote entity.

<sup>10</sup>Our modelling approach is consistent with the notion of collateral replenishment whereby non-performing encumbered assets are replaced by performing assets from the unencumbered part of the balance sheet, thereby concentrating credit and market risks on unsecured creditors.

If a proportion  $\ell \in [0, 1]$  of unsecured debt is not rolled over at  $t = 1$ , the banker liquidates an amount  $\frac{\ell UD_U}{\psi R}$  to meet withdrawals. A bank fails due to *illiquidity* at  $t = 1$  and is closed early if the liquidation value of unencumbered assets is insufficient to meet withdrawals:

$$R(1 - \alpha)I - A < \frac{\ell UD_U}{\psi}. \quad (1)$$

The decisions of fund managers exhibit strategic complementarity: an individual fund manager's incentive to roll over increases in the proportion of managers who roll over. The illiquidity threshold of the shock is  $A_{IL}(\ell) \equiv R(1 - \alpha)I - \frac{\ell UD_U}{\psi}$ .

In the event of early closure, secured debtholders are able to recover encumbered assets. But recovery may be partial, reflecting legal difficulties in seizing collateral assets (Duffie and Skeel, 2012; Ayotte and Gaon, 2011), the inability of secured debtholders to properly redeploy these assets (Diamond and Rajan, 2001), or informational losses from the disruption to bank risk management systems (Bolton and Oehmke, 2016). Accordingly, the net return for secured debtholders is  $\alpha\lambda R$ , where  $\lambda \in [\psi, 1]$  and  $1 - \lambda$  is the cost of recovering encumbered assets. Unsecured creditors are assumed to face a zero recovery rate in the event of bank failure.<sup>11</sup>

If the bank is liquid at  $t = 1$ , then the total value of bank assets is  $RI - \frac{\ell UD_U}{\psi} - A$  at  $t = 2$ . The bank fails due to *insolvency* at  $t = 2$  if it is unable to repay its secured debtholders and the proportion  $1 - \ell$  of unsecured debtholders:

$$RI - A - \frac{\ell UD_U}{\psi} < SD_S + (1 - \ell)UD_U. \quad (2)$$

Upon repaying secured debtholders at  $t = 2$ , the banker uses any residual encumbered assets (due to over-collateralization) to repay remaining unsecured debt. The

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<sup>11</sup>This assumption eases exposition but our results are qualitatively unchanged with positive recovery rates for unsecured debt.

insolvency threshold of the shock is thus  $A_{IS}(\ell) \equiv RI - SD_S - UD_U \left[ 1 + \ell \left( \frac{1}{\psi} - 1 \right) \right]$ .

At  $t = 1$ , each fund manager makes their rollover decision based on a noisy private signal about the shock. Specifically, manager  $i$  receives the signal

$$x_i \equiv A + \epsilon_i, \tag{3}$$

where  $\epsilon_i$  is idiosyncratic noise drawn from a continuous distribution  $H$  with support  $[-\epsilon, \epsilon]$  for  $\epsilon > 0$ . The idiosyncratic noise is independent of the shock, and is independently and identically distributed across fund managers.

Table 3 illustrates the timeline of events.

$t = 0$	$t = 1$	$t = 2$
1. Issuance of secured and unsecured debt	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
3. Asset encumbrance	3. Unsecured debt withdrawals	3. Debt repayments
	4. Consumption	4. Consumption

Table 3: Timeline.

### 3 Equilibrium

Our focus is on the symmetric pure-strategy perfect Bayesian equilibrium of the model. Without loss of generality, we study threshold strategies for the rollover of unsecured debt (Morris and Shin, 2003). Thus, fund managers roll over unsecured debt if, and only if, their private signals indicate a healthy balance sheet,  $x_i \leq x^*$ .

**Definition 1.** *The symmetric pure-strategy perfect Bayesian equilibrium comprises a proportion of encumbered assets ( $\alpha^*$ ), an amount of secured debt ( $S^*$ ), face values of unsecured and secured debt ( $D_U^*, D_S^*$ ), and critical thresholds for the private signal ( $x^*$ ) and balance sheet shock ( $A^*$ ) for the representative bank such that:*

- a. *at  $t = 1$ , the rollover decisions of all fund managers,  $x^*$ , are optimal and the run threshold  $A^*$  induces bank failure for any shock  $A \geq A^*$ , given the ratio of asset encumbrance and secured debt ( $\alpha^*, S^*$ ) and face values of debt ( $D_U^*, D_S^*$ );*
- b. *at  $t = 0$ , each banker optimally chooses ( $\alpha^*, S^*$ ) given the face values of debt ( $D_U^*, D_S^*$ ), the participation of secured debtholders, and the thresholds ( $x^*, A^*$ );*
- c. *at  $t = 0$ , secured and unsecured debt are priced by binding participation constraints, given the choices ( $\alpha^*, S^*$ ) and the thresholds ( $x^*, A^*$ ).*

We construct the equilibrium in four steps. First, we price secured debt. Second, we derive the optimal rollover decision of fund managers. Third, we characterize the optimal asset encumbrance choice of the banker and, in so doing, obtain the endogenous level of secured debt issuance. In a final step, we price unsecured debt.

### 3.1 Pricing secured debt

A secured debtholder either receives the face value  $D_S$  or an equal share of the value of encumbered assets. Since early closure at  $t = 1$  occurs for a large balance sheet shock, competitive pricing of secured debt by infinitely risk-averse investors implies a binding participation constraint,  $r = \min \left\{ D_S, \lambda \frac{R\alpha I}{S} \right\}$ .<sup>12</sup>

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<sup>12</sup>Incentive compatibility constraints hold if the types of investors are unobserved. A risk-averse investor strictly prefers the secured debt claim over the more volatile unsecured debt claim that can produce a total loss. A risk-neutral investor weakly prefers the unsecured debt claim. Section 6.1 analyses a risk premium between secured and unsecured debt claims.

**Lemma 1. Asset encumbrance and cheap secured debt.** *Secured debt is cheap,  $D_S^* = r$ , and the maximum issuance of secured debt tolerated by risk-averse investors is  $S \leq S^*(\alpha) = \alpha\lambda z I^*(\alpha)$ , where  $z \equiv R/r$  is the relative return and  $I^*(\alpha) = \frac{U+E}{1-\alpha\lambda z}$  is total investment. Greater encumbrance increases secured debt issuance and investment,  $\frac{dS^*}{d\alpha} = \frac{dI^*}{d\alpha} = \frac{\lambda z I^*(\alpha)}{1-\alpha\lambda z} > 0$ .*

A binding participation constraint for risk-averse investors ensures that the face value of secured debt equals the outside option of investors. Since infinitely risk-averse investors evaluate the secured debt claim at the worst outcome (the bank is closed early but encumbered assets are legally separated), the maximum level of secured debt increases in the asset encumbrance ratio. As we make clear below, a banker always chooses this maximum level of secured debt for a given encumbrance ratio, since it both reduces fragility and increases the expected equity value of the bank.

### 3.2 Rollover risk of unsecured debt

Asset encumbrance and secured debt issuance fundamentally alter the dynamics of rollover risk. Figure 2a shows the illiquidity and insolvency thresholds,  $A_{IL}(\ell)$  and  $A_{IS}(\ell)$ , without asset encumbrance and secured debt issuance,  $\alpha = S = 0$ . We recover the dynamics in Rochet and Vives (2004) for the case where liquid cash reserves are set at zero. An illiquid bank at  $t = 1$  is always insolvent at  $t = 2$ . In this case, the insolvency threshold is the relevant condition for analysis.

Figure 2b shows the illiquidity and insolvency thresholds in the case of asset encumbrance and secured debt issuance. Over-collateralization means that the thresholds do not coincide at  $\ell = 1$ . Additional assets worth  $R\alpha I^*(\alpha) - rS^*(\alpha) = R\alpha(1 - \lambda)I^*(\alpha) > 0$  become available to service unsecured debt withdrawals at

$t = 2$ , which are not available at  $t = 1$  because of encumbrance. As a result, a bank that is illiquid at  $t = 1$  can, nevertheless, be solvent at  $t = 2$ , that is  $R\alpha(1 - \lambda)I^*(\alpha) \geq (1 - \ell)UD_U$ . A sufficient condition for the illiquidity threshold to be the relevant condition for analysis is a requirement for an upper bound on the face value of unsecured debt:

$$D_U \leq \hat{D}_U \equiv (1 - \lambda)R\alpha I^*(\alpha). \quad (4)$$

We suppose that this condition holds and later verify that it does in equilibrium.

The main analysis focuses on vanishing private noise about the balance sheet shock,  $\epsilon \rightarrow 0$ , so the rollover threshold converges to the run threshold,  $x^* \rightarrow A^*$ .<sup>13</sup>

**Proposition 1. Run threshold.** *There exists a unique run threshold*

$$A^* \equiv R(1 - \alpha)I^*(\alpha) - \frac{\gamma UD_U}{\psi}. \quad (5)$$

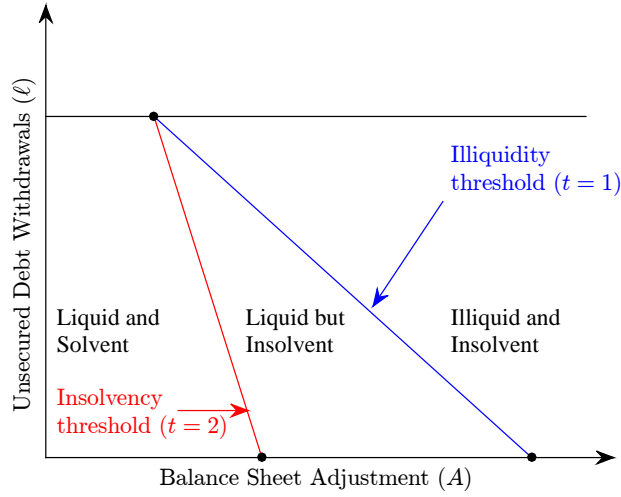
*Fund managers roll over unsecured debt at  $t = 1$  if and only if  $A \leq A^*$ , such that early closure occurs if and only if  $A > A^*$ .*

**Proof.** See Appendix A.1. ■

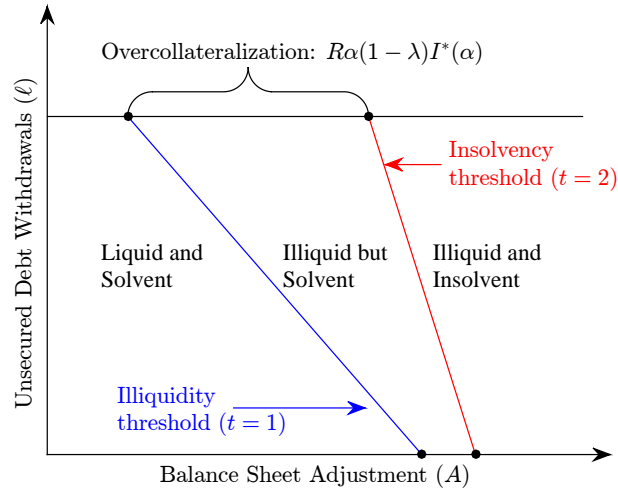
Proposition 1 uses global games techniques to pin down the unique incidence of an unsecured debt run by fund managers. More conservative fund managers decrease the threshold of the balance sheet shock above which a run occurs,  $\frac{\partial A^*}{\partial \gamma} < 0$ . A higher return on investment increases the value of unencumbered assets and the amount of secured debt raised for a given ratio of asset encumbrance. Both effects act to reduce run risk, so  $\frac{\partial A^*}{\partial R} > 0$ . A higher liquidation value of investment decreases the

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<sup>13</sup>Section 6.3 considers non-vanishing private noise and interim-date liquidity support.



(a) Without asset encumbrance ( $\alpha = S = 0$ )



(b) With asset encumbrance

Figure 2: Asset encumbrance and secured debt issuance alters the run dynamics. The figure depicts the illiquidity and insolvency thresholds as a function of the proportion of withdrawing investors  $\ell$  for two cases: without asset encumbrance in panel (a) and with asset encumbrance in panel (b). Panel (a) replicates the results of [Rochet and Vives \(2004\)](#) without liquid asset holdings and with a balance sheet shock, where the relevant condition is the insolvency threshold. In contrast, panel (b) shows that over-collateralization due to encumbrance shifts the insolvency threshold to the right. Provided the upper bound  $\hat{D}_U$  holds, the relevant condition is the illiquidity threshold.



extent of strategic complementarity among fund managers and increases the amount of secured debt issued, given encumbrance. Since both these effects also lower run risk, it follows that  $\frac{\partial A^*}{\partial \psi} > 0$ . Similarly, a decrease in the cost of recovering encumbered assets after early closure of the bank (lower  $1 - \lambda$ ) allows the banker to raise more secured funding, given encumbrance. So the stock of unencumbered assets increases and run risk decreases,  $\frac{\partial A^*}{\partial \lambda} > 0$ . A higher cost of funding decreases the amount of secured debt raised for a given encumbrance ratio. This reduces the value of unencumbered assets, so  $\frac{\partial A^*}{\partial r} < 0$ . A better capital of banks reduces run risk through its effect on increased investment and unencumbered asset values, implying  $\frac{\partial A^*}{\partial E} > 0$ .

Lemma 2 links the fragility in unsecured debt markets to secured debt issuance and the recovery of encumbered assets after early closure.

**Lemma 2. Asset encumbrance and fragility.** *If the cost of recovering encumbered assets is low,  $\lambda z > 1$ , greater asset encumbrance reduces run risk. Conversely, if  $\lambda z \leq 1$ , greater asset encumbrance heightens bank fragility:*

$$\frac{dA^*}{d\alpha}(\lambda z - 1) \geq 0, \quad (6)$$

*with strict inequality whenever  $\lambda z \neq 1$ .*

**Proof.** See Appendix A.1. ■

Greater asset encumbrance affects the run threshold in two opposing ways. First, for a given level of investment, greater asset encumbrance reduces the stock of unencumbered assets, which heightens run risk. Second, greater encumbrance allows the banker to issue more secured funding, which increases the total size of investment. As a result, the stock of unencumbered assets increases and run risk is reduced. Thus, the overall effect depends on the relative size of these two effects.

Whether the second effect dominates depends on the cost to secured debtholders of recovering encumbered assets. If the cost is high (low  $\lambda$ ) then, for each unit of secured funding raised, the banker must encumber more than one unit of assets. In this case, the stock effect dominates and greater asset encumbrance increases run risk. But if it is cheap for secured debtholders to obtain the encumbered assets (high  $\lambda$ ) then, for each unit of secured funding raised, the banker needs to encumber less than one unit of assets. Greater encumbrance accordingly lowers run risk.

**Assumption 1.** *The cost of recovering encumbered assets is high:*

$$\lambda z < 1. \tag{7}$$

In what follows, we proceed on the basis that the cost of recovering encumbered assets is high. As already noted, the inability to fully recover encumbered assets can reflect legal considerations. After the failure of a bank, secured senior debtholders frequently face costly and protracted legal proceedings with other creditors to recover collateral assets. [Duffie and Skeel \(2012\)](#) note that even secured financial instruments, such as repos, qualifying for special legal status (so-called “safe harbour” provisions) are not immune from legal logjams.<sup>14</sup> High recovery costs are also consistent with [Bolton and Oehmke \(2016\)](#), who argue that bank failure disrupts risk management and information systems, adversely affecting the realized returns to encumbered assets. While such disruptions can be mitigated by the setting up of redundant systems, such ex ante precautions are often prohibitively costly.

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<sup>14</sup>[Fleming and Sarkar \(2014\)](#) provide evidence of protracted legal settlement following the demise of Lehman’s. Although safe harbour provisions allowed retail counterparties to terminate their contracts with Lehman’s within weeks of the bankruptcy, the settlement of their claims on encumbered assets remained incomplete for several years.

### 3.3 Optimal asset encumbrance and secured debt issuance

The representative banker chooses a ratio of asset encumbrance to maximize the expected equity value of the bank, taking as given the face value of unsecured debt  $D_U$ , and subject to the run threshold,  $A = A^*(\alpha)$ , and the (maximum) amount of secured debt that can be raised,  $S \leq S^*(\alpha)$ . Since a higher level of secured debt for a given ratio of asset encumbrance both increases the expected equity value of the bank and lowers fragility, we obtain  $S = S^*(\alpha)$ . The banker's problem reduces to

$$\max_{\alpha} \pi \equiv \int E_2(A) dF(A) = \int^{A^*(\alpha)} [RI^*(\alpha) - UD_U - S^*(\alpha)r - A] dF(A). \quad (8)$$

Figure 3 shows the relationship between encumbrance and expected equity value.<sup>15</sup> A unique interior solution of asset encumbrance exists that balances the effects of asset encumbrance on the amount of secured debt raised and bank fragility.

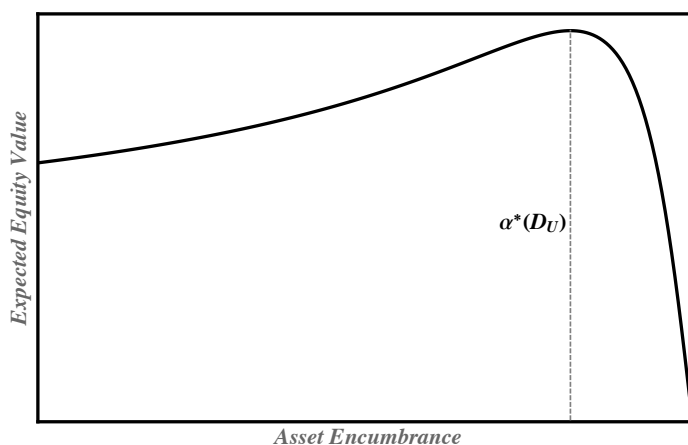


Figure 3: A unique interior privately optimal encumbrance ratio.

<sup>15</sup>For this and subsequent figures, we use the numerical example of  $R = 1.5$ ,  $r = 1.1$ ,  $E = 0.5$ ,  $\psi = 0.6$ ,  $\lambda = 0.66$ ,  $\gamma = 0.8$ ,  $D_U = 3.3$  (unless endogenous), and the balance sheet shock follows a Gaussian distribution with mean  $-3$  and unit variance.

**Proposition 2. Asset encumbrance schedule.** *There is a unique asset encumbrance schedule,  $\alpha^*(D_U)$ . If fund managers are sufficiently conservative,  $\gamma > \psi$ , then the schedule decreases in the face value of unsecured debt,  $\frac{d\alpha^*}{dD_U} \leq 0$ , and an interior solution for  $\underline{D}_U < D_U < \overline{D}_U$  is implicitly given by:*

$$\frac{F(A^*(\alpha^*))}{f(A^*(\alpha^*))} = \frac{(1 - \lambda z)}{\lambda(z - 1)} \left[ RI^*(\alpha^*)\alpha^*(1 - \lambda) + UD_U \left( \frac{\gamma}{\psi} - 1 \right) \right]. \quad (9)$$

**Proof.** See Appendix A.2. ■

The banker balances the marginal benefits and costs of asset encumbrance when choosing the privately optimal encumbrance ratio. The marginal benefit of encumbrance is an increase in the amount of secured funding obtained. Since secured debt is cheap and investments are profitable, the equity value of the bank – conditional on surviving an unsecured debt run – is higher. But the marginal cost of encumbrance is an increase in bank fragility and, therefore, a lower probability of surviving an unsecured debt run. So a higher face value of unsecured debt exacerbates rollover risk and lowers the run threshold, inducing the banker to encumber fewer assets.

### 3.4 Pricing of unsecured debt

The repayment of unsecured debt depends on the size of the balance sheet shock. In the absence of a run,  $A \leq A^*$ , unsecured debtholders receive the promised payment  $D_U$ , while for larger shocks,  $A > A^*$ , bankruptcy occurs and they receive zero. The value of an unsecured debt claim is  $V(D_U, \alpha) \equiv D_U F(A^*(\alpha, D_U))$  and competitive pricing implies that it equals the cost of funding for any given encumbrance ratio:

$$r = D_U^* F(A^*(\alpha, D_U^*)). \quad (10)$$

**Proposition 3. Private optimum.** *If bank capital is scarce,  $E < \bar{E}$ , and managers are conservative,  $\gamma \geq \underline{\gamma}$ , then there exists a unique face value of unsecured debt,  $D_U^* > r$ . If funding is costly,  $r > \underline{r}$ , asset encumbrance is interior,  $\alpha^{**} \equiv \alpha^*(D_U^*) \in (0, 1)$ .*

**Proof.** See Appendix A.3. ■

Figure 4 shows the privately optimal allocation and its construction. The condition  $\gamma \geq \underline{\gamma}$  ensures that the schedule  $D_U^*(\alpha)$ , which is derived from the market clearing condition for the face value of unsecured debt, is upward-sloping in the vicinity of the asset encumbrance schedule. This, in turn, leads to a unique characterization of the joint equilibrium for the ratio of asset encumbrance,  $\alpha^{**}$ , and the face value of unsecured debt,  $D_U^{**}$ . Next, the condition that the bank's capital must satisfy  $E < \bar{E}$  ensures that  $D_U^* < \hat{D}_U(\alpha^*)$ , and that the illiquidity condition is more binding than the insolvency condition for the bank, as supposed. Finally, the lower bound on the cost of funding ensures that the equilibrium face value of unsecured debt is sufficiently high, such that the equilibrium ratio of asset encumbrance is interior.

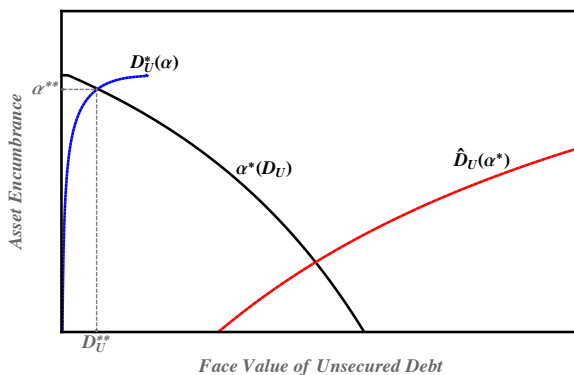


Figure 4: Privately optimum of asset encumbrance and face value of unsecured debt.

The allocation in Proposition 3 is constrained efficient. Our welfare criterion is constrained efficiency, namely the allocation chosen by the social planner taking as

given the incomplete information structure of the game (that is, the private information of fund managers about balance sheet adjustment). Constrained efficiency arises as the pricing of debt is competitive and there are no externalities across banks.<sup>16</sup>

## 4 Testable implications

Our model yields a rich set of comparative static results. Parameter changes affect the unique interior equilibrium in two ways. First, for a given face value of unsecured debt, the banker trades off heightened fragility against more profitable investment funded with cheap secured debt. Second, the equilibrium face value of unsecured debt changes with underlying parameter values, influencing the required face value for investors to participate in the unsecured debt contract. We summarize the main results in Proposition 4, before discussing each of them in turn.

**Proposition 4.** *The banker’s privately optimal ratio of asset encumbrance,  $\alpha^{**}$ , decreases in the cost of funding,  $r$ , conservatism of fund managers,  $\gamma$ , and the costs of recovering encumbered assets,  $1 - \lambda$ . Encumbrance increases in the profitability of investment,  $R$ , improvements in the shock distribution  $F(\cdot)$ , and the liquidation value,  $\psi$ . Greater bank capital,  $E$ , has an ambiguous effect on asset encumbrance.*

**Proof.** See Appendix A.4. ■

**Monetary policy.** A lower cost of funding, perhaps brought about by easier monetary conditions, increases secured debt issuance and increases the benefits of asset encumbrance. Since the required face value of unsecured debt is also lowered, the two effects combine to increase asset encumbrance,  $\frac{d\alpha^{**}}{dr} < 0$ .

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<sup>16</sup>We study the normative implications of a modified model with guarantees in section 5.

Our model suggests that less restrictive monetary conditions lead to a shift toward secured lending. This implication appears consistent with the stylized evidence in the wake of the global financial crisis. Since 2007/8, central banks in advanced countries have run expansionary monetary policy and there has been an increased appetite for safe assets among investors (IMF, 2013; Caballero et al., 2016). Consistent with this implication, Juks (2012) and Bank of England (2012) document a clear, increasing trend in the encumbrance ratios of Swedish and UK banks following the implementation of extraordinary monetary policy measures in response to the crisis.

**Profitability.** Higher returns on investment or a more favorable distribution of the balance sheet shock – in the sense of a first-order stochastic dominance shift according to the reverse hazard rate – reduces fragility and induces the banker to encumber more. Therefore, the model implies that banks with less risky balance sheets or more profitable assets increase the share of secured debt on their balance sheet. Consistent with these implications, DiFilippo et al. (2016) document that banks with higher risks reduce their share of secured debt.

**Liquidation values.** Higher liquidation values decrease the degree of strategic complementarity among fund managers for a given ratio of asset encumbrance, reducing illiquidity and fragility at  $t = 1$ . Hence, the banker encumbers more assets and increases profitable investment. Lower fragility, in turn, reduces the face value of unsecured debt required for investors to participate, increasing encumbrance further.<sup>17</sup>

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<sup>17</sup>In the context of mutual funds, Chen et al. (2010) identify strategic complementarities in their investments and show that illiquid funds face greater strategic complementarities than liquid funds because of the higher costs associated with redemptions.

**Recovery costs.** A decrease in the cost of recovering encumbered assets has two effects. First, for each unit of secured funding, the bankers has to encumber fewer assets. So for a given ratio of asset encumbrance, the banker can raise more secured funding. Second, there are more unencumbered assets available to meet withdrawals at  $t = 1$ , which reduces fragility. This, in turn, lowers the face value of unsecured debt. Taking both effects into account, there is greater asset encumbrance,  $\frac{d\alpha^{**}}{d\lambda} > 0$ .

**Market stress.** The conservatism parameter  $\gamma$  can be broadly interpreted as a measure of market stress. [Krishnamurthy \(2010\)](#) documents how, during the global financial crisis, fund managers turned conservative and became less inclined to roll over unsecured debt. Faced with a deterioration in investor sentiment, the bank is more fragile for a given encumbrance ratio. The banker responds to the heightened fragility in a precautionary fashion, lowering the extent of encumbrance and forgoing profitable investment from the issuance of secured debt in order to induce rollovers by fund managers. The combined effects of increased fragility and the greater face value of unsecured debt reduce encumbrance, so  $\frac{d\alpha^{**}}{d\gamma} < 0$ . Increased market stress thus induces a reduction in the share of secured debt (as a proportion of total debt).

**Capital buffers.** Our model suggests that the effect of an increase in bank capital on asset encumbrance is ambiguous. There are two opposing effects. On the one hand, more capital enables the bank to withstand larger balance sheet shocks and so lowers fragility. While this “loss absorption” effect induces greater encumbrance, the bank risks losing more of its own funds in bankruptcy. The result of such “greater skin in the game” is to lower encumbrance. [Figure 5](#) shows how these two opposing effects induce a non-monotonic relationship between bank capital and asset encumbrance. [Lemma 3](#) suggests tighter predictions can be made with specific distributional assumptions.



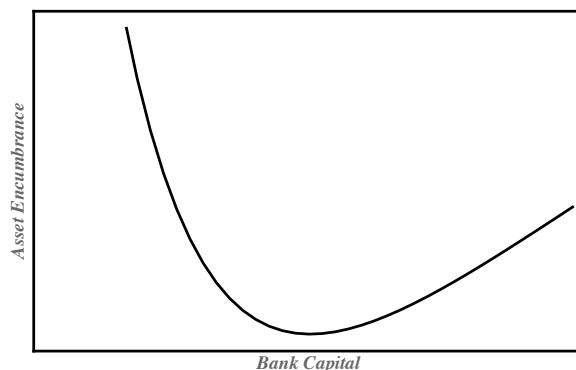


Figure 5: A non-monotonic relationship between bank capital and encumbrance. The relationship between the capital ratio and encumbrance (not plotted) is also non-monotonic.

**Lemma 3.** *If the balance sheet shock is uniformly distributed, then the privately optimal ratio of asset encumbrance increases in bank capital.*

**Proof.** See Appendix A.5. ■

## 5 Policy implications

In many countries, unsecured debtholders often enjoy the benefits of explicit (or implicit) government guarantees. These schemes, which typically apply to retail depositors, often extend to wholesale depositors during times of crisis. Although deposit insurance and wholesale funding guarantee schemes have been studied previously, their relationship with asset encumbrance has yet to be examined.<sup>18</sup> Specifically, while guarantee schemes aim to reduce bank fragility ex post, their presence distorts behavior ex ante. By externalizing the costs to the guarantor upon failure, bankers have incentives to excessively encumber assets that, in turn, creates excessive fragility.

<sup>18</sup>Earlier contributions include, Kareken and Wallace (1978); Diamond and Dybvig (1983); Calomiris (1990); Matutes and Vives (1996), and Cooper and Ross (2002).

Our model provides a natural framework for the normative analysis of this issue. Let a fraction  $0 < m < 1$  of unsecured debt be fully guaranteed and the guarantor (i.e., the government) be deep-pocketed.<sup>19</sup> Withdrawing guaranteed debt at  $t = 1$  is a weakly dominated for unsecured debtholders. Let the face value of guaranteed unsecured debt be  $D_G$ . If the fraction  $\ell$  of unsecured non-guaranteed debt is withdrawn at  $t = 1$ , then the bank is illiquid whenever

$$R(1 - \alpha)I - A \leq \frac{\ell(1 - m)UD_U}{\psi}, \quad (11)$$

where the guarantee reduces the illiquidity of the banker at  $t = 1$  for given funding choices. Similarly, the bank is insolvent at  $t = 2$  whenever

$$RI - A - \frac{\ell(1 - m)UD_U}{\psi} \leq SD_S + (1 - \ell)(1 - m)UD_U + mUD_G, \quad (12)$$

where the guarantee reduces the insolvency of the banker at  $t = 2$ , since guaranteed unsecured debt is not subject to rollover risk at  $t = 1$  and cheaper in any equilibrium.

The equilibrium in secured debt and guaranteed unsecured debt is as follows. Backed by the guarantor, guaranteed debt is safe,  $D_G^* = r$ , in any equilibrium. The balance sheet adjustment has full support, so the bank is closed early at  $t = 1$  with positive probability for any given guarantee coverage. Moreover, since the guarantee has no bearing on how encumbered assets are managed following early closure, Lemma 1 continues to hold, implying  $D_S^* = r$  and  $S^* = S^*(\alpha)$  and  $I^* = I^*(\alpha)$ .<sup>20</sup>

<sup>19</sup>In the spirit of Allen et al. (2015), we consider guarantees that eliminate both inefficient and efficient runs. As the authors argue, such a guarantee scheme resembles real-world deposit insurance.

<sup>20</sup>The upper bound on the face value of unsecured debt is more relaxed with guarantees. Thus, the conditions previously imposed continue to suffice for the illiquidity threshold to be more binding than the insolvency threshold, irrespective of guarantee coverage. To see this, the bound is

$$D_U \leq \hat{D}_U(m) \equiv \frac{1}{1 - m} \left[ \frac{R\alpha(1 - \lambda)I^*(\alpha)}{U} - mr \right]. \quad (13)$$

Figure 6 shows how introducing the guarantee affects run dynamics. The illiquidity and insolvency thresholds, respectively, depend on the volume of withdrawals:

$$A_{IL}(\ell) = R(1 - \alpha)I^*(\alpha) - \ell \frac{(1 - m)UD_U}{\psi}, \quad (14)$$

$$A_{IS}(\ell) = R(1 - \alpha\lambda)I^*(\alpha) - UD_U(1 - m) \left( 1 + \ell \left[ \frac{1}{\psi} - 1 \right] \right). \quad (15)$$

Guarantees reduces the the responsiveness of both thresholds to changes in the withdrawal volume, with a stronger effect on the illiquidity constraint. Moreover, the insolvency threshold shifts outwards, while the illiquidity threshold pivots outwards.

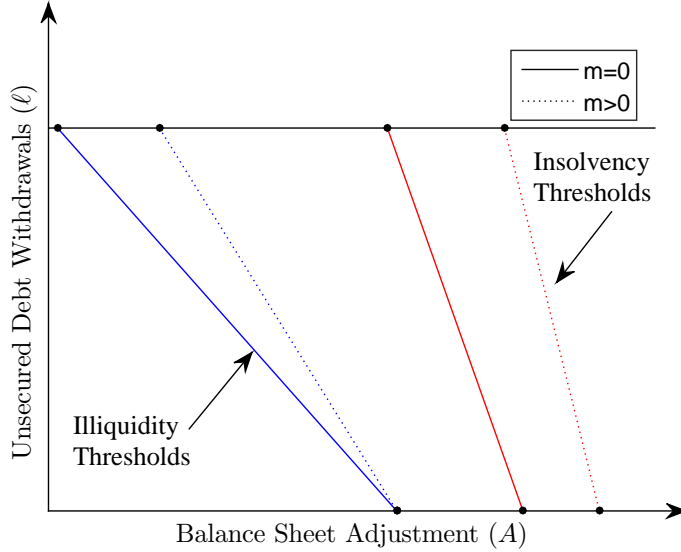


Figure 6: The impact of guarantee coverage on the run dynamics. The figure shows the illiquidity and insolvency thresholds as a function of the withdrawal volume  $\ell$  before (solid lines) and after (dashed lines) the introduction of the guarantee.

Paralleling the previous arguments, and maintaining vanishing private noise, a funds manager withdraws unsecured debt without guarantee at  $t = 1$  if and only if

$$A > A_m^* \equiv R(1 - \alpha)I^*(\alpha) - \frac{\gamma(1 - m)UD_U}{\psi}. \quad (16)$$

---

A higher coverage of the guarantee relaxes the condition in equation (13), since  $\partial \hat{D}_U(m) / \partial m = (\hat{D}_U(m) - r) / (1 - m) \geq 0$  because  $D_U \geq r$  in any equilibrium.

The direct effect of the guarantee is to increase the run threshold and thus to decrease the incidence of early closure,  $\frac{\partial A_m^*}{\partial m} = \frac{\gamma U D_U}{\psi} > 0$ , which is the intended purpose of introducing a guarantee. However, the introduction of a guarantee also affects the incentives of the banker to encumber assets and, as a result, the pricing of unsecured debt. The banker's problem in the presence of the guarantee is:

$$\alpha_m^* \equiv \max_{\alpha} \pi_m(\alpha) = \int^{A_m^*(\alpha)} [R I^*(\alpha) - S^*(\alpha) r - (1-m) U D_U - m U r - A] dF(A). \quad (17)$$

**Proposition 5. Privately optimal encumbrance with guarantees.** *If the coverage of the guarantee satisfies  $m \leq \bar{m}$ , then there exists a unique equilibrium for the ratio of asset encumbrance and face value of unsecured debt. The presence of guarantees increases the ratio of asset encumbrance,  $\alpha_m^{**} > \alpha^{**}$ .*

**Proof.** See Appendix A.6. ■

The presence of a guarantee has two intuitive effects. First, the stock of unsecured debt that may be withdrawn at  $t = 1$  is reduced, which lowers the incidence of runs. As a result, the banker has more incentives to encumber assets, which shifts the encumbrance schedule  $\alpha_m^*(D_U)$  outward. Second, unsecured debt without a guarantee is repaid with greater probability, since the run threshold increases. This, in turn, reduces the face value of unsecured debt and shifts the participation constraint of investors  $D_U^*(\alpha)$  inward. In sum, the introduction of the guarantee unambiguously increases the encumbrance ratio,  $\alpha_m^{**}$ , but has an ambiguous effect on the equilibrium face value of unsecured debt,  $D_m^{**} \equiv D_U^*(\alpha_m^{**})$ .

The banker, however, does not internalize the cost of servicing the guarantee. These costs are instead incurred by the guarantor, who must contribute the amount

$mUr$  upon failure of the bank, which occurs with probability  $1 - F(A_m^*)$ .<sup>21</sup> The planner's asset encumbrance schedule for a given face value of unsecured debt,  $\alpha_P^*(D_U)$ , accounts for the expected cost of providing the guarantee.<sup>22</sup>

$$\alpha_P^*(D_U) \equiv \max_{\alpha} \pi_m(\alpha) - [1 - F(A_m^*(\alpha))] mUr. \quad (18)$$

While the asset encumbrance schedules of the banker and planner differ, the participation constraint of unsecured debtholders is the same and given by the same participation constraint schedule for investors,  $\tilde{D}_U^*(\alpha)$ . So any difference in allocations is due to differences in the respective asset encumbrance schedules. The face value of unsecured debt chosen by the planner, denoted by  $D_P^{**}$ , solves the fixed point problem  $D_P^{**} \equiv D_U^*(\alpha_P^*(D_U^*))$ . Figure 7 shows the asset encumbrance schedules and the pricing of unsecured non-guaranteed debt, as well as the privately and socially optimal ratios of asset encumbrance and face values of debt.

**Proposition 6. Constrained inefficiency.** *The decentralized equilibrium with guarantees is constrained inefficient. The privately optimal asset encumbrance and the face value of unsecured non-guaranteed debt are excessive,  $\alpha_m^{**} > \alpha_P^{**}$  and  $D_m^{**} > D_P^{**}$ .*

**Proof.** See Appendix A.7. ■

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<sup>21</sup>As stated in the introduction, we abstract from explicitly modeling premiums for deposit insurance (DI). Since these premiums are typically insensitive to encumbrance ratios, a bank still has an incentive to encumber assets excessively even if premiums are otherwise fairly priced. See also Proposition 7 and our interpretation of the tax as an encumbrance surcharge to the DI premium.

<sup>22</sup>Our welfare analysis abstracts from the payoffs to fund managers. Using the payoffs proposed by Rochet and Vives (2004), these are  $bF(A^*)$  in equilibrium. Because of vanishing private noise, each fund manager refuses to roll over unsecured debt exactly when the bank fails at  $t = 1$ . Accounting for these payoffs, however, would *increase* the gap between the privately and socially optimal ratios of encumbrance, since the probability of bank survival is lower under excessive encumbrance. Therefore, our results on the welfare-enhancing effect of regulation are reinforced.

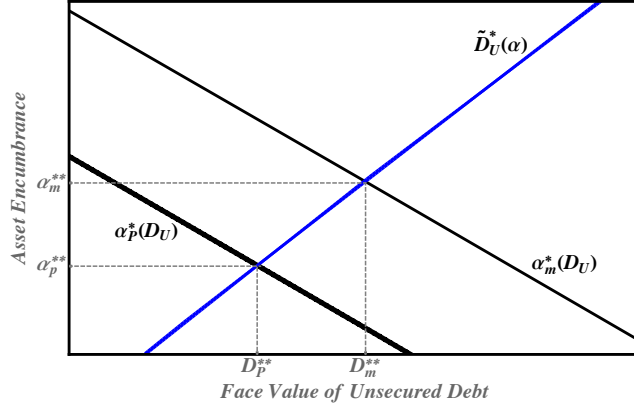


Figure 7: Constrained inefficiency after the introduction of the guarantee scheme: the socially optimal asset encumbrance schedule lies below the privately optimal schedule for any given face value of debt,  $\alpha_P^*(D_U) \leq \alpha_m^*(D_U)$ . Since the competitive pricing of unsecured and non-guaranteed debt is the same for the planner and the banker, the privately optimal ratios of asset encumbrance and face value of debt are excessive.

**Prudential safeguards.** We now consider several policy tools to curb excessive encumbrance and restore constrained efficiency in the presence of guarantees. In particular, we consider a limit to asset encumbrance,  $\alpha \leq \bar{\alpha}$  as an additional constraint to the bankers problem. We consider policies that are less information-intensive for the policymaker, notably lump-sum taxes and transfers,  $T$ , and linear, revenue-neutral Pigovian taxes on asset encumbrance. Let  $\alpha_R^*(D_U)$  denote the banker's optimal asset encumbrance schedule subject to some regulation.

**Proposition 7. Optimal regulation.** *In the presence of the guarantee,  $m > 0$ , a planner achieves the constrained efficient allocation,  $(\alpha_P^{**}, D_P^{**})$ , by imposing:*

- a. a limit on asset encumbrance at  $\alpha_P^{**}$ ;
- b. a transfer  $T = Umr$  to the banker at  $t = 2$ ;
- c. a contingent linear tax on asset encumbrance imposed at  $t = 2$ , combined with a lump-sum rebate of the generated revenue,  $T = \alpha\tau$ . The optimal rate is

$$\tau^*(D_U) = \frac{(1 - \lambda z)RI^*(\alpha_P^*)Umr f(A_m^*(\alpha_P^*))}{(1 - \lambda z\alpha_P^*)F(A_m^*(\alpha_P^*))}, \quad (19)$$

which depends on the the face value of unsecured non-guaranteed debt.

- d. a linear tax on asset encumbrance at  $t = 2$  that is not contingent on the face value of debt, combined with a lump-sum rebate of the generated revenue. That is, there exists a unique rate  $\tau^* > 0$  such that  $\alpha_R^{**}(\tau^*) = \alpha_P^{**}$  and  $D_R^{**}(\tau^*) = D_P^{**}$ .

**Proof.** See Appendix A.8. ■

Figure 8 shows how a limit on asset encumbrance achieves constrained efficiency. When the constraint  $\alpha \leq \alpha_P^{**}$  is added to the banker's problem, the banker's constrained encumbrance schedule for any given face value of debt is

$$\alpha_R^*(D_U) \equiv \begin{cases} \alpha_m^*(D_U) & \alpha_m^*(D_U) < \alpha_P^{**} \\ \alpha_P^{**} & \alpha_m^*(D_U) \geq \alpha_P^{**}. \end{cases} \quad \text{if} \quad (20)$$

The constrained banker chooses the efficient encumbrance ratio and, therefore, unsecured debt is also priced at the efficient level. In sum, when the planner directly controls encumbrance at  $t = 0$ , a limit on encumbrance achieves constrained efficiency.

The encumbrance cap can also be implemented via bank capital regulation. The bank's capital ratio at  $t = 0$  (inverse leverage ratio),  $e \equiv e(\alpha) = \frac{E_0}{I^*(\alpha)}$ , is sensitive to changes in the encumbrance ratio, specifically  $\frac{de}{d\alpha} < 0$ . Thus, a minimum capital ratio,  $e \geq \underline{e}$ , translates into a bound on asset encumbrance in our model. Thus, constrained efficiency can be achieved by requiring that the bank's capital ratio satisfies  $e > \underline{e}(\alpha_P^{**})$ . This argument generalizes to risk-based capital requirements if encumbered and unencumbered assets carry different risk-weights.

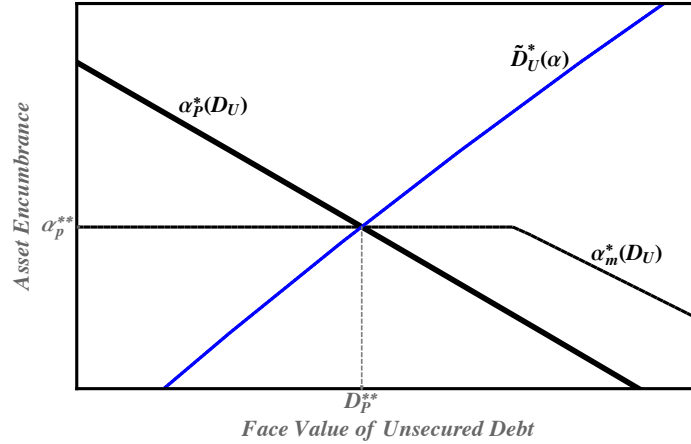


Figure 8: A limit on asset encumbrance achieves constrained efficiency.

We proceed to consider tools that do not require the planner to observe the encumbrance ratio at  $t = 0$ . Figure 9 shows how a transfer to the banker at  $t = 2$  achieves constrained efficiency. When the banker receives the market value of non-guaranteed unsecured debt,  $Umr$ , it internalizes the effect of its failure on the guarantor, that is the social cost of bank failure. A benefit of this transfer is that the planner does not need to observe the encumbrance ratio at all. However, the size of this transfer may be huge and its implementation subject to political constraints.

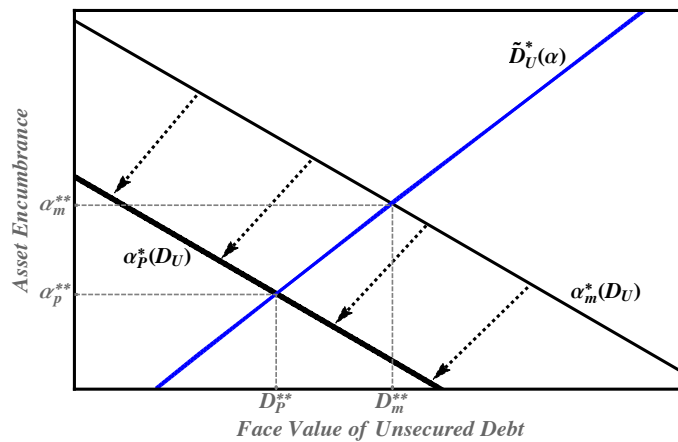


Figure 9: A transfer of the market value of guaranteed unsecured debt achieves constrained efficiency by aligning the socially and privately optimal asset encumbrance schedules. That is, it induces the banker to internalize the social cost of its failure.



We next turn to revenue-neutral regulation. Specifically, we consider a linear tax  $\tau$  on asset encumbrance imposed at  $t = 2$ , combined with a lump-sum rebate of  $T = \tau\alpha$ . A higher tax rate reduces the privately optimal encumbrance of assets, since the banker's equity value contingent upon no early closure decreases in encumbrance. When the tax rate can be contingent on the face value of unsecured non-guaranteed debt, the optimal rate  $\tau^*(D_U)$  ensures that the privately and socially optimal asset encumbrance schedules are aligned for any face value of unsecured debt. This situation parallels that of a transfer in Figure 9. As a result, constrained efficiency is attained.

When the tax rate cannot be contingent on the face value of debt, the privately and socially optimal encumbrance schedules cannot generically be aligned, as shown in Figure 10. However, a higher tax rate reduces asset encumbrance, so there exists a unique rate at which the constrained efficient allocation is achieved. Basically, the privately optimal asset encumbrance schedule has to be shifted in by the marginal tax on encumbrance by enough to cross through the constrained efficient allocation.

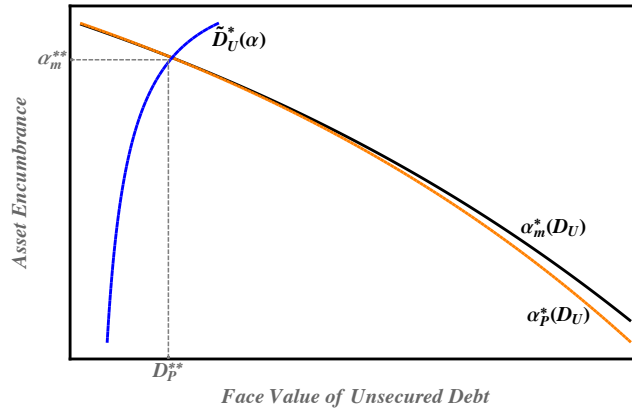


Figure 10: A non-contingent linear tax on encumbrance with lump-sum rebate achieves constrained efficiency.

In our analysis, the linear Pigovian tax and its lump-sum rebate are imposed at  $t = 2$ . Since this requires the planner to only observe asset encumbrance at this date, it can be viewed as a less information-intensive policy tool. The timing assumption

is without loss of generality, however. If a tax and lump-sum rebate scheme were imposed at  $t = 1$ , the banker would internalize the effect of asset encumbrance on heightened illiquidity and thus a greater incidence of early closure, which induces a banker to encumber fewer assets.<sup>23</sup> If a tax and lump-sum rebate scheme were imposed at  $t = 0$ , the available resources for investment would be affected. The banker would internalize the effect of great encumbrance on reduced profitable investment, which also exacerbates illiquidity at  $t = 1$ . In sum, the full rebate of the tax revenue generated ensures that the actual investment size at  $t = 0$ , actual illiquidity at  $t = 1$ , or actual equity value at  $t = 2$  is unchanged, while aligning the banker's incentives.

**Relation to policy debate.** The problem of excessive asset encumbrance by banks has become a growing focus of attention for policymakers. Banks have increasingly turned to secured funding in a quest for safety that may, ultimately, be counterproductive. Policymakers have expressed concern that the increased collateralization of bank balance sheets can heighten rollover risk and, hence, bank fragility, as well as increase the procyclicality of the financial system (Haldane, 2012; CGFS, 2013).

The prudential safeguards considered in this section speak to some of these concerns. Policymakers in some countries are seeking to address excessive encumbrance by imposing explicit restrictions on the share of bank assets that can be encumbered. These restrictions apply either (a) through limits on assets that can be pledged when secured debt is issued; or (b) via limits on bond issuance. In the context of our model, these asset and liability side restrictions are equivalent and map directly into the cap on asset encumbrance. Table 4 summarizes the restrictions implemented.

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<sup>23</sup>The credibility of a regulatory measure perceived to heighten fragility at  $t = 1$ , precisely when a bank may become illiquid, is a matter of debate, however.

Country	Policies	
	Assets	Liabilities
Australia	8%	
Belgium	8%	
Canada	4%	
Greece	20%	
Italy	$\begin{cases} 25\% \text{ of assets if } 6\% \leq CET1 < 7\% \\ 60\% \text{ of assets if } 7\% \leq CET1 < 8\% \\ \text{No Limit} & \text{if } CET1 \geq 8\% \end{cases}$	
Netherlands	Determined on a case-by-case basis, such that the ratio of encumbered to total assets must be a ‘healthy ratio’	
New Zealand	10%	
United States		4%

Table 4: Caps on asset encumbrance across different countries

While most countries have adopted a cap on encumbered assets, the United States has implemented a policy to cap the share of secured debt to total liabilities. In Italy, the asset encumbrance cap is contingent on a bank’s Core Equity Tier 1 capital ratio (CET1). However, as we have shown, there is a non-linear relationship between asset encumbrance and bank capital (and also capital ratio). So the effects of such a policy for fragility may not be clear-cut.

In our model, the cap on asset encumbrance is set at the level that maximizes the social planner’s objective function. But the cap can, in practice, be sensitive to broader economic and financial conditions. So, to maintain a constrained efficient outcome, policy-makers may need to consider asset encumbrance policies that vary over time. In the Netherlands, for example, the cap is set on a case-by-case basis for individual banks, taking into account the financial position, solvency risk of the issuing bank, its risk profile and the riskiness inherent in its assets (see [DNB, 2015](#)).

A form of Pigovian taxation to address the externalities posed by asset encumbrance has also been implemented in Canada. The deposit insurance premiums levied by the Canadian Deposit Insurance Corporation (CDIC, 2017) on systemically important domestic banks reflects the extent to which their balance sheet is encumbered. Specifically, 5% of the score used to calculate the deposit insurance premium paid by these banks reflects asset encumbrance considerations. Such a surcharge can be viewed as being similar in spirit to the asset encumbrance tax analyzed above, although the extent to which the CDIC surcharge is revenue-neutral is open to debate.

## 6 Additional Implications

We conclude the analysis of the model by considering the implications of (i) a risk premium between risk-neutral and risk-averse investors; (ii) a limited wealth constraint for risk-averse investors; and (iii) the precision of private information about the balance sheet shock.

### 6.1 Risk premium on unsecured debt

The required return for risk-averse and risk-neutral investors has been the same so far. We now suppose that risk-neutral investors require a risk premium. Formally, investors have access to a menu of risk-free and risky storage. The return on risk-free storage continues to be  $r$ , while the expected return on risky storage is  $\tilde{r} > r$ , implying a risk premium of  $p \equiv \tilde{r} - r$ . We derive an additional testable implication about the risk premium. The risk premium does not affect the pricing of secured debt and the illiquidity condition, so  $D_S^* = r$ ,  $S^*(\alpha)$ , and  $A^*(\alpha)$  are unchanged. Since the banker takes the face value of unsecured debt,  $D_U$ , as given, the privately optimal ratio of

asset encumbrance,  $\alpha^*(D_U)$ , is also unchanged. But the pricing of unsecured debt is affected, as the competitive pricing schedule  $D_U^*(\alpha; p)$  shifts out as the risk premium increases. Figure 11 illustrates and Proposition 8 summarizes.

**Proposition 8.** *A higher risk premium,  $p$ , decreases the privately optimal ratio of asset encumbrance,  $\alpha^{**}$ , and increases the face value of unsecured debt,  $D_U^{**}$ .*

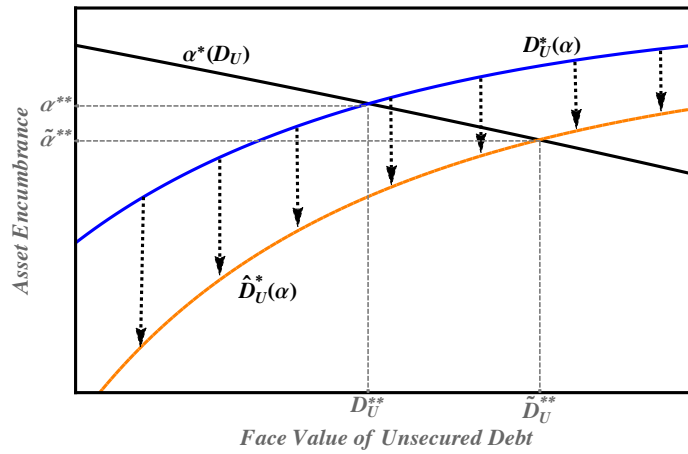


Figure 11: A higher risk premium reduces asset encumbrance and increases the face value of unsecured debt.

## 6.2 Limited wealth

In our model, the banker trades off more profitable investment, funded by cheap secured debt, against greater fragility when choosing asset encumbrance. This mechanism relies on the pool of risk-averse investors being sufficiently large, such that the banker can obtain secured funding up to the level where the marginal benefit from an additional unit of cheap funding is equal to the marginal cost of greater fragility. This trade-off, however, can break down once the wealth of risk-averse investors is limited. In what follows, we investigate this case formally.

Suppose that a mass  $\omega$  of risk-averse investors has a unit endowment. So the supply of funds by risk-averse investors, given by their binding participation constraint, is:

$$D_S^* \equiv \begin{cases} r & \alpha \leq \bar{\alpha} \equiv \frac{\omega}{\lambda z(U+E+\omega)} \\ \text{if} & \\ r \frac{\lambda z(U+E+\omega)}{\omega} \alpha & \alpha > \bar{\alpha}, \end{cases} \quad (21)$$

where  $\bar{\alpha}$  is the proportion of total investment at which the total wealth of risk-averse investors is attracted with secured debt. The amount of secured debt raised is:

$$S^*(\alpha) \equiv \begin{cases} \alpha \lambda z I^*(\alpha) & \alpha \leq \bar{\alpha} \\ \text{if} & \\ \omega & \alpha > \bar{\alpha}. \end{cases} \quad (22)$$

As before, the cost of greater asset encumbrance is greater fragility, whereby  $\frac{dA^*}{d\alpha} < 0$ . However, there is no longer a benefit of greater asset encumbrance when the wealth of risk-averse investors is scarce, since no additional secured debt can be issued. Moreover, the equilibrium face value of secured debt increases in asset encumbrance, further reducing the expected equity value of the banker as asset encumbrance increases. Therefore, it is never optimal to encumber more assets than  $\bar{\alpha}$ .

### 6.3 Limited precision of information and liquidity support

In the limit of infinitely precise private information about the balance sheet shock,  $\epsilon \rightarrow 0$ , the mass of fund managers who withdraw is a step function,  $\ell^*(A, x) = \mathbf{1}_{\{\mathbf{A} > \mathbf{A}^*\}}$ . If the bank's insolvency and illiquidity lines are sufficiently close, then  $\ell^*(A, x)$  crosses both curves for the same threshold level  $A^*$ , implying that the illiquidity and insolvency thresholds are the same. For finite precision, by contrast, the mass of fund

managers who withdraw is  $\ell^*(A, x^*) = H(x^* - A)$ , which is a sigmoidal function. Consequently, the illiquidity and insolvency thresholds differ (Figure 12).

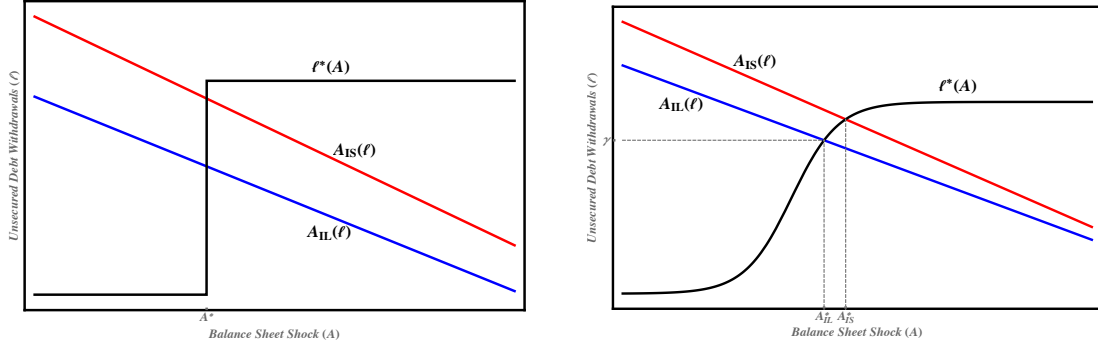


Figure 12: The precision of private information and the range of balance sheet shocks for which a bank is illiquid but solvent. The top panel shows the case of infinitely precise private information and  $D_U \geq \frac{\psi}{1-\gamma} \hat{D}_U$ , so  $A_{IL}^{**} = A_{IS}^{**}$ . The bottom panel shows the case of limited precision of private information and the range  $[A_{IL}^{**}, A_{IS}^{**}]$ .

For values of the balance sheet shock in the wedge between the illiquidity and insolvency thresholds, the bank is indeed illiquid but solvent. This opens up a role for a lender of last-resort, as considered by [Rochet and Vives \(2004\)](#). In particular, if the central bank observes the shock without noise – perhaps due to its supervisory function – it can offer loans at an interest rate  $\rho \in [0, \frac{1}{\psi} - 1)$ . Such a lender of last resort policy shifts out the illiquidity threshold for any given encumbrance ratio. Since the bank is truly solvent, no taxpayer’s money is at risk. An alternative support mechanism is costly liquidity injections ([Cong et al., 2017](#)). In the context of our model, such an intervention shifts up the illiquidity threshold by an amount  $\phi$  at an exogenous cost  $k(\phi)$ . With credible commitment, such a policy balances the cost with the endogenous benefit of reducing both fragility and the cost of unsecured debt. Both forms of liquidity support affect the banker’s incentives to encumber assets at  $t = 0$ . Detailed consideration of this issue is, however, left for future research.

## 7 Conclusion

Banks increasingly rely on secured funding attracted by encumbering assets: a bank ring-fences and legally separates some assets on its balance sheet into a bankruptcy-remote vehicle. In this paper, we offer a model of asset encumbrance by banks subject to rollover risk and examine its effect on the fragility and funding costs of banks. The privately optimal encumbrance ratio trades off expanding profitable investment funded by cheap secured debt with greater fragility due to unsecured debt runs. We derive and discuss several testable implications about the encumbrance ratios and secured funding shares of banks. Deposit insurance and wholesale funding guarantees induce excessive encumbrance and exacerbates fragility. To mitigate these risk-shifting incentives, we show how prudential regulation of banks should be designed.



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# A Proofs

## A.1 Proof of Proposition 1

The proof is in three steps. First, we show that the dominance regions at the rollover stage, based on the illiquidity threshold, are well defined for any bank funding structure. If the balance sheet shock were common knowledge, the rollover behavior of fund managers would be characterized by multiple equilibria, as shown in Figure 13. If no unsecured debt is rolled over,  $\ell = 1$ , the bank is liquid at  $t = 1$  whenever the shock is below a lower dominance bound  $\underline{A} \equiv R(1 - \alpha)I - \frac{UDU}{\psi}$ . For  $A < \underline{A}$ , it is a dominant strategy for fund managers to roll over. If  $\ell = 0$ , the bank is illiquid whenever the shock is above an upper dominance bound  $\bar{A} \equiv R(1 - \alpha)I$ . For  $A > \bar{A}$ , it is a dominant strategy for managers not to roll over.

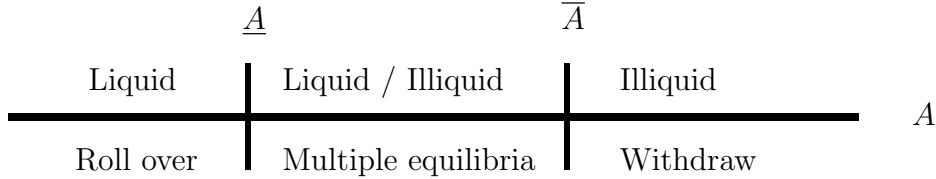


Figure 13: Tripartite classification of the balance sheet shock

Second, in any rollover stage, it suffices to establish the optimality of threshold strategies for sufficiently precise private information. [Morris and Shin \(2003\)](#) and [Frankel et al. \(2003\)](#) show that only threshold strategies survive the iterated deletion of strictly dominated strategies. Thus, each fund manager  $i$  uses a threshold strategy, whereby unsecured debt is rolled over if and only if the private signal about the balance sheet shock is below some signal threshold,  $x_i < x^*$ . We consider vanishing private noise,  $\epsilon \rightarrow 0$ , in the main text.

Third, we characterize this threshold equilibrium. For a given realization  $A \in [\underline{A}, \bar{A}]$ , the proportion of fund managers who do not roll over unsecured debt is:

$$\ell(A, x^*) = \text{Prob}(x_i > x^* | A) = \text{Prob}(\epsilon_i > x^* - A) = 1 - H(x^* - A). \quad (23)$$

A **critical mass condition** states that illiquidity occurs when the shock equals  $A^*$ , where the proportion of managers not rolling over is evaluated at  $A^*$ :

$$R(1 - \alpha)I - A^* = \ell(A^*, x^*) \frac{UD_U}{\psi}. \quad (24)$$

The posterior distribution of the shock conditional on the private signal is derived using Bayes' rule. An **indifference condition** states that the manager who receives the threshold signal  $x_i = x^*$  is indifferent between rolling and not rolling over,  $\gamma = \Pr(A < A^* | x_i = x^*)$ . Using the definition  $x_j = A + \epsilon_j$ , the conditional probability is

$$1 - \gamma = \Pr(A \geq A^* | x_i = x^*) = \Pr(A \geq A^* | x_i = x^* = A + \epsilon_j), \quad (25)$$

$$= \Pr(x^* - \epsilon_j \geq A^*) = \Pr(\epsilon_j \leq x^* - A^*) = H(x^* - A^*). \quad (26)$$

The indifference condition implies  $x^* - A^* = H^{-1}(1 - \gamma)$ . Inserting this expression into  $\ell(A^*, x^*)$ , the proportion of managers who do not roll over at the threshold shock level  $A^*$  is perceived by the indifferent manager to be  $\ell(A^*, x_i = x^*) = 1 - H(x^* - A^*) = 1 - H(H^{-1}(1 - \gamma)) = \gamma$ . The run threshold  $A^*$  stated in Proposition 1 follows. It varies with the face value of unsecured debt and asset encumbrance according to:

$$\frac{dA^*}{dD_U} = -\frac{\gamma U}{\psi} < 0, \quad \frac{dA^*}{d\alpha} = R(\lambda z - 1) \frac{I^*(\alpha)}{1 - \alpha \lambda z}. \quad (27)$$

## A.2 Proof of Proposition 2

The banker's problem is given in (8). The total derivative  $\frac{d\pi}{d\alpha}$ , which takes indirect effects via  $A^*(\alpha)$  and  $S^*(\alpha)$  into account, yields  $\frac{d\pi}{d\alpha} = \frac{RI^*(\alpha)}{1 - \alpha \lambda z} f(A^*) G(\alpha)$ , where

$$G(\alpha) = \frac{F(A^*)}{f(A^*)} \lambda(z - 1) - (1 - \lambda z) \left[ RI^*(\alpha) \alpha (1 - \lambda) + UD_U \left( \frac{\gamma}{\psi} - 1 \right) \right]. \quad (28)$$

If an interior solution  $0 < \alpha^* < 1$  exists, it is given by  $G(\alpha^*) = 0$ . It is a local maximum:

$$\frac{dG}{d\alpha} = \underbrace{\frac{dF(A^*)}{f(A^*)} \frac{dA^*}{d\alpha}}_{+} \lambda(z-1) - (1-\lambda z) \frac{R(1-\lambda)I^*(\alpha)}{1-\alpha\lambda z} < 0, \quad (29)$$

where the first term is positive since  $f(A)$  possesses decreasing reverse hazard rate. Using the implicit function theorem (IFT), we obtain  $\frac{d\alpha^*}{dD_U} < 0$  for the interior solution since

$$\frac{dG}{dD_U} = \frac{dF(A^*)}{f(A^*)} \frac{dA^*}{dD_U} \lambda(z-1) - (1-\lambda z)U \left( \frac{\gamma}{\psi} - 1 \right) < 0 \quad (30)$$

where we used  $\gamma > \psi$ . Hence, the weak inequality  $\frac{d\alpha^*}{dD_U} \leq 0$  follows for the entire schedule of asset encumbrance. We next examine if the solution to  $G(\alpha^*) = 0$  is interior. An interior solution requires two conditions,  $G(\alpha = 0) > 0$  and  $G(\alpha = 1) < 0$ , which we consider in turn. First, evaluating the implicit function when there is no encumbrance,  $\alpha = 0$ , yields

$$\frac{F(A^*(0))}{f(A^*(0))} \lambda(z-1) - (1-\lambda z) \left( \frac{\gamma}{\psi} - 1 \right) U D_U, \quad (31)$$

which strictly decreases in  $D_U$ . To ensure  $\alpha^* > 0$ , the face value of unsecured debt must satisfy  $D_U < \bar{D}_U$ , where  $\bar{D}_U$  is uniquely and implicitly defined by

$$\frac{F(R(U+E) - \frac{\gamma}{\psi} U \bar{D}_U)}{f(R(U+E) - \frac{\gamma}{\psi} U \bar{D}_U)} \lambda(z-1) - (1-\lambda z) \left( \frac{\gamma}{\psi} - 1 \right) U \bar{D}_U = 0. \quad (32)$$

Second, evaluating the implicit function when all assets are encumbered,  $\alpha = 1$ , yields

$$\frac{F(A^*(1))}{f(A^*(1))} \lambda(z-1) - (1-\lambda z) \left[ \frac{R(1-\lambda)(U+E)}{1-\lambda z} + \left( \frac{\gamma}{\psi} - 1 \right) U D_U \right], \quad (33)$$

which also decreases in  $D_U$ . To ensure  $\alpha^* < 1$ , the expression in equation (33) must be strictly negative. Hence, the face value of unsecured debt must be bounded from below,  $D_U > \underline{D}_U$ , where  $\underline{D}_U < \bar{D}_U$  is uniquely and implicitly defined by

$$\frac{F(-\frac{\gamma}{\psi} U \underline{D}_U)}{f(-\frac{\gamma}{\psi} U \underline{D}_U)} \lambda(z-1) - (1-\lambda z) \left[ \frac{R(1-\lambda)(U+E)}{1-\lambda z} + \left( \frac{\gamma}{\psi} - 1 \right) U \underline{D}_U \right] = 0. \quad (34)$$

### A.3 Proof of Proposition 3

The proof is in five steps. First, we ensure that the encumbrance ratio is interior,  $\alpha^{**} \in (0, 1)$ . Since  $\alpha = 0$  implies  $\hat{D}_U = 0$ , there is no equilibrium consistent with our supposition  $D_U \leq \hat{D}_U$ , so it must be the case that  $\alpha^{**} > 0$  (we verify the supposition below). If  $\alpha = 1$ , the run threshold and value of an unsecured debt claim are  $A^*(1) = -\frac{\gamma D_U}{\psi}$  and  $V(1, D_U) = D_U F(A^*(1))$ , respectively. The value of the unsecured debt claim attains a maximum at  $D_U = D_{max}$ , which is uniquely and implicitly defined by  $\frac{F(-\frac{\gamma}{\psi} D_{max})}{f(-\frac{\gamma}{\psi} D_{max})} - \frac{\gamma}{\psi} D_{max} = 0$ . Since  $V(\alpha, D_U)$  decreases in  $\alpha$ , if we impose on the outside option that  $r > \underline{r} \equiv V(1, D_{max})$ , then any solution for the equilibrium ratio of asset encumbrance is interior (if it exists).

Second, we show that the equilibrium face value of unsecured debt satisfies  $D_U^* > r$ . To this end, note that while  $\frac{\partial V}{\partial D_U}$  has an ambiguous sign in general, the derivative evaluated along the asset encumbrance schedule is

$$\left. \frac{\partial V}{\partial D_U} \right|_{\alpha^*(D_U)} = f(A^*) \left[ \frac{1 - \lambda z}{\lambda(z - 1)} R(1 - \lambda) \alpha^* I^*(\alpha^*) + \beta_0 U D_U \right], \quad (35)$$

which is non-negative whenever  $\beta_0 \equiv \frac{1 - \lambda z}{\lambda(z - 1)} \left( \frac{\gamma}{\psi} - 1 \right) - \frac{\gamma}{\psi} \geq 0 \leftrightarrow \gamma \geq \underline{\gamma} \equiv \frac{1 - \lambda z}{1 - \lambda z - \lambda(z - 1)} \psi$ . Having established conditions under which  $V$  increases in  $D_U$ , at least in the vicinity of the asset encumbrance schedule, it follows that  $D_U = r$  always violates the participation constraint,  $V(D_U = r) = r F(A^*(D_U = r)) < r$ . Thus,  $D_U^* > r$ .

Third, we establish that the intersection between the asset encumbrance schedule and participation constraint of unsecured debtholders yields a unique joint equilibrium. Proposition 2 states that  $\alpha^*(D_U)$  decreases in  $D_U$ . Also, from the second step of this proof, we have sufficient conditions that ensure, in the vicinity of the the asset encumbrance schedule, the market-implied face value of unsecured debt,  $D_U^*(\alpha)$ , increases in  $\alpha$ . Hence, there is at most only one intersection of these two curves, establishing equilibrium uniqueness.

Fourth, we show that the equilibrium specified above exists. Define  $T(D_U) = r / F(A^*(\alpha^*(D_U), D_U))$



as a mapping from the set  $\mathcal{U}$  of face values of unsecured debt into itself. If  $\mathcal{U}$  is a closed and compact set then, by Brouwer's fixed-point theorem, there exists at least one fixed-point for the mapping. The lower bound on  $D_U$  is  $r$ . For the upper bound, note that if the banker could pledge all assets to unsecured investors, then  $D_U \leq RI^*(\alpha) - A$ . Truncating the shock distribution at some arbitrary  $-A_L < 0$  yields a well-defined upper bound on  $D_U$ .

Fifth, we verify the supposition  $D_U \leq \hat{D}_U \equiv \alpha RI^*(\alpha)(1-\lambda)$ . Denoting the run threshold, evaluated at  $D_U = \hat{D}_U$ , by  $\hat{A}^*(\alpha) \equiv A^*(\hat{D}_U(\alpha))$ , we have  $\hat{A}^*(\alpha) = RI^*(\alpha) \left[ 1 - \alpha \left( 1 + \frac{\gamma}{\psi}(1-\lambda) \right) \right]$  with

$$\frac{d\hat{A}^*}{d\alpha} = -\frac{RI^*(\alpha)}{1-\alpha\lambda z} \left[ 1 - \lambda z + \frac{\gamma}{\psi}(1-\lambda) \right] < 0. \quad (36)$$

Next, define  $\hat{\alpha}^*$  as the equilibrium ratio of asset encumbrance evaluated at  $D_U = \hat{D}_U(\hat{\alpha}^*)$ :

$$\frac{F(\hat{A}^*(\hat{\alpha}^*))}{f(\hat{A}^*(\hat{\alpha}^*))} = \frac{1-\lambda z}{\lambda(z-1)} \frac{\gamma(1-\lambda)}{\psi} R\hat{\alpha}^* I^*(\hat{\alpha}^*), \quad (37)$$

which implicitly defines  $\hat{\alpha}^*$ . Since the left-hand side decreases in  $\alpha$ , while the right-hand side increases in it, it follows that  $\hat{\alpha}^*$  is uniquely defined. We now translate the condition  $D_U \leq \hat{D}_U$  into a condition for unsecured debt pricing:

$$r \leq V(\hat{\alpha}^*, \hat{D}_U(\hat{\alpha}^*)). \quad (38)$$

A stricter sufficient condition is to require  $r \leq V(1, \hat{D}_U(1))$ . Using the condition for  $\hat{\alpha}^*$ , we obtain

$$r \leq \frac{\psi\lambda(z-1)}{\gamma(1-\lambda z)} \frac{F\left(-\frac{R(1+E)}{1-\lambda z} \frac{\gamma}{\psi}(1-\lambda)\right)^2}{f\left(-\frac{R(1+E)}{1-\lambda z} \frac{\gamma}{\psi}(1-\lambda)\right)}, \quad (39)$$

where right-hand side decreases in the bank's initial capital,  $E$ . This suggests that imposing an upper bound,  $E \leq \bar{E}$ , on the bank's capital will ensure that, in equilibrium,  $D_U^* \leq \hat{D}_U(\alpha^*)$ . The upper bound on capital is implicitly defined as  $\frac{\psi\lambda(z-1)}{\gamma(1-\lambda z)} \frac{F\left(-\frac{R(1+\bar{E})}{1-\lambda z} \frac{\gamma}{\psi}(1-\lambda)\right)^2}{f\left(-\frac{R(1+\bar{E})}{1-\lambda z} \frac{\gamma}{\psi}(1-\lambda)\right)} = r$ .

## A.4 Proof of Proposition 4

The proof is in two steps. First, we show the effect of a parameter on the encumbrance schedule  $\alpha^*(D_U)$ . Second, we show that this direct effect is reinforced by an indirect effect via the equilibrium cost of unsecured debt  $D_U^*$ . For the direct effect via  $\alpha^*(D_U)$ , we take  $D_U$  as given and use the IFT, whereby  $\frac{d\alpha^*}{dy} = -\frac{\frac{dG}{dy}}{\frac{dG}{d\alpha}}$  for  $y \in \{\gamma, r, R, \psi, E, F(\cdot)\}$ :

$$\frac{dG}{d\gamma} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*} \frac{dA^*}{d\gamma} \lambda(z-1) - (1-\lambda z)U \frac{D_U}{\psi} < 0, \quad (40)$$

$$\begin{aligned} \frac{dG}{dr} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*} \frac{dA^*}{dr} \lambda(z-1) - \frac{F(A^*)}{f(A^*)} \lambda \frac{z}{r} \\ &\quad - \lambda \frac{z}{r} \left[ \frac{R(1-\lambda)\alpha(1-\alpha)I^*(\alpha)}{1-\alpha\lambda z} + D_U \left( \frac{\gamma}{\psi} - 1 \right) \right] < 0 \end{aligned} \quad (41)$$

$$\begin{aligned} \frac{dG}{dR} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*} \frac{dA^*}{dR} \lambda(z-1) + \frac{F(A^*)}{f(A^*)} \frac{\lambda}{r} \\ &\quad + \frac{\lambda}{r} D_U \left( \frac{\gamma}{\psi} - 1 \right) + (1-\lambda)\alpha I^*(\alpha) \left\{ \lambda z - \frac{(1-\lambda z)}{1-\alpha\lambda z} \right\}. \end{aligned} \quad (42)$$

The expression in equation (42) has an ambiguous sign in general. Therefore, we evaluate this derivative at  $\alpha^*$  by substituting  $F(A^*)/f(A^*)$  from the first-order condition in equation (9). Grouping terms, we obtain that  $dG/dR > 0$ . Moreover, we have

$$\frac{dG}{d\psi} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*} \frac{dA^*}{d\psi} \lambda(z-1) + (1-\lambda z)U D_U \frac{\gamma}{\psi^2} > 0. \quad (43)$$

The derivatives with respect to  $\lambda$  and  $E$  yield:

$$\begin{aligned} \frac{dG}{d\lambda} &= \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*} \frac{dA^*}{d\lambda} \lambda(z-1) + \frac{F(A^*)}{f(A^*)} (z-1) + zRI^*(\alpha)\alpha(1-\lambda) \\ &\quad + zUD_U \left( \frac{\gamma}{\psi} - 1 \right) + \frac{(1-\lambda z)(1-\alpha z)}{1-\alpha\lambda z} R\alpha I^*(\alpha) > 0, \end{aligned} \quad (44)$$

$$\frac{dG}{dE} = \frac{d\frac{F(A^*)}{f(A^*)}}{dA^*} \frac{dA^*}{dE} \lambda(z-1) - \frac{1-\lambda z}{1-\alpha\lambda z} R(1-\lambda)\alpha \stackrel{\leq}{>} 0. \quad (45)$$

Finally, suppose that the balance sheet shock distribution  $\tilde{F}$  first-order stochastically dominates the distribution  $F$  according to the reverse hazard rate. This implies that

$$\frac{\tilde{f}}{\tilde{F}} \geq \frac{f}{F}, \quad (46)$$

or, equivalently,  $F/f \geq \tilde{F}/\tilde{f}$ . Let  $\tilde{G}(\tilde{\alpha}^*) = 0$  denote the implicit function defining the privately optimal asset encumbrance ratio,  $\tilde{\alpha}^*$ , under the balance sheet shock distribution  $\tilde{F}$ . Thus,  $\tilde{G}(\alpha) \leq G(\alpha)$  for all ratios of encumbrance. Since  $d\tilde{G}/d\alpha < 0$  and  $dG/d\alpha < 0$ , the privately optimal encumbrance ratio satisfy  $\tilde{\alpha}^* \geq \alpha^*$ .

The indirect effects arise from the equilibrium face value of unsecured debt. For any given encumbrance ratio, they are given by the implicit function  $J(\alpha, D_U^*) = 0$  where

$$J \equiv -r + D_U F(A^*(\alpha, D_U)) \quad (47)$$

Using the implicit function again, and noting that  $\left. \frac{\partial J}{\partial D_U} \right|_{\alpha^*} > 0$ , we obtain reinforcing effects:

$$\frac{\partial J}{\partial R} = D_U f(A^*) \frac{dA^*}{dR} > 0, \quad \frac{\partial J}{\partial \psi} = D_U f(A^*) \frac{dA^*}{d\psi} > 0 \quad (48)$$

$$\frac{\partial J}{\partial \lambda} = D_U f(A^*) \frac{dA^*}{d\lambda} > 0, \quad \frac{\partial J}{\partial \gamma} = D_U f(A^*) \frac{dA^*}{d\gamma} < 0 \quad (49)$$

$$\frac{\partial J}{\partial E} = D_U f(A^*) \frac{dA^*}{dE} > 0, \quad \frac{\partial J}{\partial r} = -1 + D_U f(A^*) \frac{dA^*}{dr} < 0. \quad (50)$$

Finally, an improvement in the distribution of the balance sheet shock also increases  $J$ .

## A.5 Proof of Lemma 3

If the balance sheet shock is uniformly distributed in the interval  $[-A_L, A_H]$ , then the equilibrium encumbrance ratio, for a given  $D_U$  is

$$R I^*(\alpha^*) \left[ 1 - \alpha^* \left\{ 1 + \frac{(1-\lambda)(1-\lambda z)}{\lambda(z-1)} \right\} \right] = U D_U \left\{ \frac{\gamma}{\psi} + \left( \frac{\gamma}{\psi} - 1 \right) \frac{(1-\lambda z)}{\lambda(z-1)} \right\} - A_L, \quad (51)$$

which uniquely defines  $\alpha^*$ . It follows that  $d\alpha^*/dE > 0$  by the IFT, since the left-hand side of equation (51) decreases in the encumbrance ratio and increases in bank capital.

## A.6 Proof of Proposition 5

The derivation for the private optimum with guarantees follows closely that in Appendix A.3. For brevity, we only state the key conditions for the existence of a unique equilibrium.

**Banker's asset encumbrance schedule.** Taking the derivative of the banker's objective function with respect  $\alpha$ , the optimal encumbrance ratio, for a given face value of unsecured debt, is implicitly defined by  $G_m(\alpha_m^*(D_U)) = 0$ , where

$$G_m(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)} \lambda(z-1) - (1-\lambda z) \left[ R\alpha(1-\lambda)I^*(\alpha) + (1-m)UD_U \left( \frac{\gamma}{\psi} - 1 \right) - mUr \right]$$

and  $\frac{dG_m}{d\alpha} < 0$ , so the solution is a local maximum. As before, for an interior solution,  $\alpha_m^*(D_U) \in (0, 1)$ , we require that  $D_U \in (\underline{D}_U(m), \bar{D}_U(m))$ . For the rest of the normative analysis, we assume that the encumbrance schedule yields interior solutions that are local maxima. Bounds similar to those above can be derived analogously to the previous section.

### Participation constraint of unsecured and non-guaranteed debtholders.

The value of the unsecured debt with a guarantee satisfies, in equilibrium,  $r = V(\alpha, D_U, m) \equiv D_U^* F(A_m^*(\alpha, D_U^*))$ . Following the lines of reasoning previously established, we obtain that a joint equilibrium exists if (i)  $r > \underline{r}_m \equiv V(1, D_{max}, m)$ , (ii)  $\gamma \geq \underline{\gamma} \equiv \frac{1-\lambda z}{1-\lambda z - \lambda(z-1)}\psi$ , (iii)  $m \leq \bar{m} \equiv \frac{\frac{1-\lambda z}{\lambda(z-1)}\left(\frac{\gamma}{\psi}-1\right)-\frac{\gamma}{\psi}}{\left(\frac{\gamma}{\psi}-1\right)\left[\frac{1-\lambda z}{\lambda(z-1)}-1\right]}$ , and (iv)  $E \leq \bar{E}_m$ .

**Comparative statics.** We derive how the encumbrance ratio depends on the coverage of the guarantee. As before, we derive separately the direct effect on the asset encumbrance schedule and the indirect effect from the face value of unsecured debt. For the direct effect,

the derivative of the implicit function,  $G_m(\alpha)$  with respect to coverage is

$$\frac{dG_m}{dm} = \frac{d\frac{F(A_m^*)}{f(A_m^*)}}{dA_m^*} \frac{dA_m^*}{dm} \lambda(z-1) + (1-\lambda z)U \left[ D_U \left( \frac{\gamma}{\psi} - 1 \right) + r \right] > 0. \quad (52)$$

Since we have established  $dG_m/d\alpha < 0$ , it follows from the IFT that  $d\alpha_m^*/dm > 0$  for any given  $D_U$ , so the encumbrance schedule shifts outwards following an increase in coverage.

The indirect effect concerns the face value of unsecured debt given by the implicit function  $J_m(\alpha, D_U^*(m)) = 0$ , where  $J_m \equiv -r + D_U F(A_m^*(\alpha, D_U))$ . Using the implicit function, note that  $\left. \frac{dJ_m}{dD_U} \right|_{\alpha^*} > 0$ , so  $\frac{dJ_m}{dm} = D_U f(A_m^*) \frac{dA_m^*}{dm} > 0$ . Hence, the face value of unsecured debt is reduced as coverage increases, which reinforces the encumbrance incentives.

## A.7 Proof of Proposition 6

Taking the derivative of the planner's objective function with respect to the encumbrance ratio, we obtain the first-order condition  $\frac{d\pi_m}{d\alpha} + f(A_m^*) \frac{dA_m^*}{d\alpha} mUr = 0$ . The planner's asset encumbrance schedule,  $\alpha_P^*(D_U)$ , is given by  $G_P(\alpha_P^*) = 0$ , where

$$G_P(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)} \lambda(z-1) - (1-\lambda z) \left[ R\alpha(1-\lambda)I^*(\alpha) + (1-m)UD_U \left( \frac{\gamma}{\psi} - 1 \right) \right], \quad (53)$$

which decreases in  $\alpha$ . We again focus on the interior solutions. Comparing equation (53) to the implicit function that provides the banker's encumbrance schedule in equation (52), we have that  $G_P(\alpha_m^*) < 0$  for all permissible  $D_U$ . Hence,  $\alpha_m^*(D_U) > \alpha_P^*(D_U)$  for any  $m$ .

## A.8 Proof of Proposition 7

With a limit on asset encumbrance, the banker's constrained problem is given by

$$\alpha_m^* \equiv \max_{\alpha \in [0, \alpha_P^{**}]} \pi_m(\alpha) = \int^{\alpha_m^*(\alpha)} [RI^*(\alpha) - (1-m)UD_U - S^*(\alpha)r - mUr - A] dF. \quad (54)$$

Since the bank profit is concave around  $\alpha_m^*$ , the marginal profit at  $\alpha = \alpha_P^{**}$  is strictly positive. The constrained optimum of encumbrance stated in equation (20) follows.

For the tax and transfer schemes, suppose that the planner imposes a linear tax  $\tau > 0$  on asset encumbrance at  $t = 2$  combined with a lump-sum transfer  $T$ . Then, the privately optimal asset encumbrance schedule subject to this regulation is given by the implicit function  $G_R(\alpha)$ , where  $\alpha_R^*(D_U)$  solves  $G_R(\alpha_R^*(D_U)) \equiv 0$ :

$$G_R(\alpha) \equiv \frac{F(A_m^*)}{f(A_m^*)} \lambda(z-1) - \frac{F(A_m^*)}{f(A_m^*)} \frac{1-\alpha\lambda z}{RI^*(\alpha)} \tau + \dots \quad (55)$$

$$-(1-\lambda z) \left[ R\alpha(1-\lambda)I^*(\alpha) - \tau\alpha + T + (1-m)UD_U \left( \frac{\gamma}{\psi} - 1 \right) - mUr \right].$$

Consider the pure transfer scheme,  $\tau = 0$ . Comparing  $G_R(\alpha)|_{\tau=0}$  with  $G_P(\alpha)$  in equation (53), the two asset encumbrance schedules are the same whenever  $T = Umr$ .

Next, consider a revenue-neutral scheme,  $T = \tau\alpha$  for all  $\alpha$ . We evaluate  $\alpha_R^*(D_U)$  at  $T = \tau\alpha$  and solve for the optimal tax. Equalizing the socially optimal and the privately optimal schedule under regulation,  $\alpha_R^* = \alpha_P^*$ , we obtain  $\tau^*$  stated in equation (19). This tax rate depends on the face value of unsecured and non-guaranteed debt.

Finally, we consider a linear tax on encumbrance independent of  $D_U$ . Using the IFT, we obtain that  $\frac{d\alpha_R^*}{d\tau} < 0$  since  $\frac{dG_R}{d\alpha} < 0$  by optimality and

$$\frac{dG_R}{d\tau} \Big|_{T=\tau\alpha} = -\frac{1-\alpha\lambda z}{RI^*(\alpha)} \frac{F(A_m^*)}{f(A_m^*)} < 0. \quad (56)$$